Little Harmonic Labyrinth

The Tortoise and Achilles are spending a day at Coney Island.

After buying a couple of cotton candies, they decide to take a ride

on the Ferris wheel.

Tortoise: This is my favorite ride. One seems to move so far, and yet in reality one gets nowhere.

Achilles: I can see why it would appeal to you. Are you all strapped in? Tortoise: Yes, I think I've got this buckle done. Well, here we go. Whee!

Achilles: You certainly are exuberant today.

Tortoise: I have good reason to be. My aunt, who is a fortune-teller, told me that a stroke of Good Fortune would befall me today. So I am tingling with anticipation.

Achilles: Don't tell me you believe in fortune-telling!

Tortoise: No ... but they say it works even if you don't believe in it.

Achilles: Well, that's fortunate indeed.

Tortoise: Ah, what a view of the beach, the crowd, the ocean, the city . . .

Achilles: Yes, it certainly is splendid. Say, look at that helicopter over there. It seems to be flying our way. In fact it's almost directly above us now.

Tortoise: Strange—there's a cable dangling down from it, which is coming very close to us. It's coming so close we could practically grab it.

Achilles: Look! At the end of the line there's a giant hook, with a note.

(He reaches out and snatches the note. They pass by and are on their way down.)

Tortoise: Can you make out what the note says?

Achilles: Yes—it reads, "Howdy, friends. Grab a hold of the hook next time around, for an Unexpected Surprise."

Tortoise: The note's a little corny but who knows where it might lead. Perhaps it's got something to do with that bit of Good Fortune due me. By all means, let's try it!

Achilles: Let's!

(On the trip up they unbuckle their buckles, and at the crest of the ride, they grab for the giant hook. All of a sudden they are whooshed up by the cable, which quickly reels them skyward into the hovering helicopter. A large strong hand helps them in.)

Voice: Welcome aboard—Suckers.

Achilles: Wh—who are you?

Voice: Allow me to introduce myself. I am Hexachlorophene J. Goodfortune, Kidnapper-At-Large, and Devourer of Tortoises par Excellence, at your service.

Tortoise: Gulp!

Achilles (whispering to his friend): Uh-oh—I think that this "Goodfortune" is not exactly what we'd anticipated. (To Goodfortune) Ah—if I may be so bold—where are you spiriting us off to?

Goodfortune: Ho ho! To my all-electric kitchen-in-the-sky, where I will prepare THIS tasty morsel—(leering at the Tortoise as he says this)—in a delicious pie-in-the-sky! And make no mistake—it's all just for my gobbling pleasure! Ho ho ho!

Achilles: All I can say is you've got a pretty fiendish laugh.

Goodfortune (laughing fiendishly): Ho ho ho! For that remark, my friend, you will pay dearly. Ho ho!

Achilles: Good grief-I wonder what he means by that!

Goodfortune: Very simple—I've got a Sinister Fate in store for both of you! Just you wait! Ho ho ho! Ho ho ho!

Achilles: Yikes!

Goodfortune: Well, we have arrived. Disembark, my friends, into my fabulous all-electric kitchen-in-the-sky.

(They walk inside.)

Let me show you around, before I prepare your fates. Here is my bedroom. Here is my study. Please wait here for me for a moment. I've got to go sharpen my knives. While you're waiting, help yourselves to some popcorn. Ho ho ho! Tortoise pie! Tortoise pie! My favorite kind of pie! (Exit.)

Achilles: Oh, boy-popcorn! I'm going to munch my head off!

Tortoise: Achilles! You just stuffed yourself with cotton candy! Besides, how can you think about food at a time like this?

Achilles: Good gravy—oh, pardon me—I shouldn't use that turn of phrase, should I? I mean in these dire circumstances . . .

Tortoise: I'm afraid our goose is cooked.

Achilles: Say—take a gander at all these books old Goodfortune has in his study. Quite a collection of esoterica: Birdbrains I Have Known; Chess and Umbrella-Twirling Made Easy; Concerto for Tapdancer and Orchestra . . . Hmmm.

Tortoise: What's that small volume lying open over there on the desk, next to the dodecahedron and the open drawing pad?

Achilles: This one? Why, its title is Provocative Adventures of Achilles and the Tortoise Taking Place in Sundry Spots of the Globe.

Tortoise: A moderately provocative title.

Achilles: Indeed—and the adventure it's opened to looks provocative. It's called "Djinn and Tonic".

Tortoise: Hmm... I wonder why. Shall we try reading it? I could take the Tortoise's part, and you could take that of Achilles.

Achilles: I'm game. Here goes nothing ...

(They begin reading "Djinn and Tonic".)

(Achilles has invited the Tortoise over to see his collection of prints by his favorite artist, M. C. Escher.)

Tortoise: These are wonderful prints, Achilles.

Achilles: I knew you would enjoy seeing them. Do you have any particular favorite?

Tortoise: One of my favorites is Convex and Concave, where two internally consistent worlds, when juxtaposed, make a completely inconsistent composite world. Inconsistent worlds are always fun places to visit, but I wouldn't want to live there.

Achilles: What do you mean, "fun to visit"? Inconsistent worlds don't EXIST, so how can you visit one?

Tortoise: I beg your pardon, but weren't we just agreeing that in this Escher picture, an inconsistent world is portrayed?

Achilles: Yes, but that's just a two-dimensional world—a fictitious world—a picture. You can't visit that world.

Tortoise: I have my ways ...

Achilles: How could you propel yourself into a flat picture-universe?

Tortoise: By drinking a little glass of PUSHING-POTION. That does the trick.

Achilles: What on earth is pushing-potion?

Tortoise: It's a liquid that comes in small ceramic phials, and which, when drunk by someone looking at a picture, "pushes" him right into the world of that picture. People who aren't aware of the powers of pushing-potion often are pretty surprised by the situations they wind up in.

Achilles: Is there no antidote? Once pushed, is one irretrievably lost?

Tortoise: In certain cases, that's not so bad a fate. But there is, in fact, another potion—well, not a potion, actually, but an elixir—no, not an elixir, but a—a—

Tortoise: He probably means "tonic".

Achilles: Tonic?

Tortoise: That's the word I was looking for! "POPPING-TONIC" is what it's called, and if you remember to carry a bottle of it in your right hand as you swallow the pushing-potion, it too will be pushed into the picture; then, whenever you get a hankering to "pop" back out into real life, you need only take a swallow of popping-tonic, and presto! You're back in the real world, exactly where you were before you pushed yourself in.

Achilles: That sounds very interesting. What would happen if you took some popping-tonic without having previously pushed yourself into a picture?

Tortoise: I don't precisely know, Achilles, but I would be rather wary of horsing around with these strange pushing and popping liquids. Once I had a friend, a Weasel, who did precisely what you suggested—and no one has heard from him since.

Achilles: That's unfortunate. Can you also carry along the bottle of pushing-potion with you?

Tortoise: Oh, certainly. Just hold it in your left hand, and it too will get pushed right along with you into the picture you're looking at.

Achilles: What happens if you then find a picture inside the picture which you have already entered, and take another swig of pushing-potion?

Tortoise: Just what you would expect: you wind up inside that picture-in-a-picture.

Achilles: I suppose that you have to pop twice, then, in order to extricate yourself from the nested pictures, and re-emerge back in real life.

Tortoise: That's right. You have to pop once for each push, since a push takes you down inside a picture, and a pop undoes that.

Achilles: You know, this all sounds pretty fishy to me . . . Are you sure you're not just testing the limits of my gullibility?

Tortoise: I swear! Look—here are two phials, right here in my pocket. (Reaches into his lapel pocket, and pulls out two rather large unlabeled phials, in one of which one can hear a red liquid sloshing around, and in the other of which one can hear a blue liquid sloshing around.) If you're willing, we can try them. What do you say?

Achilles: Well, I guess, ahm, maybe, ahm . . .

Tortoise: Good! I knew you'd want to try it out. Shall we push ourselves into the world of Escher's Convex and Concave?

Achilles: Well, ah, . . .

Tortoise: Then it's decided. Now we've got to remember to take along this flask of tonic, so that we can pop back out. Do you want to take that heavy responsibility, Achilles?

Achilles: If it's all the same to you, I'm a little nervous, and I'd prefer letting you, with your experience, manage the operation.

Tortoise: Very well, then.

(So saying, the Tortoise pours two small portions of pushingpotion. Then he picks up the flask of tonic and grasps it firmly in his right hand, and both he and Achilles lift their glasses to their lips.)

Tortoise: Bottoms up!

(They swallow.)

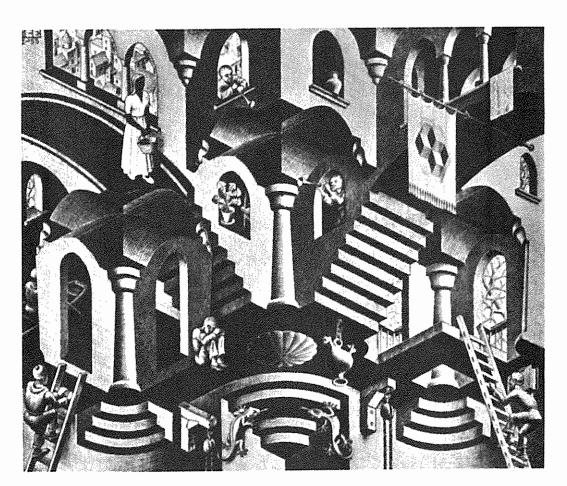


FIGURE 23. Convex and Concave, by M. C. Escher (lithograph, 1955).

Achilles: That's an exceedingly strange taste.

Tortoise: One gets used to it.

Achilles: Does taking the tonic feel this strange?

Tortoise: Oh, that's quite another sensation. Whenever you taste the tonic, you feel a deep sense of satisfaction, as if you'd been waiting to taste it all your life.

Achilles: Oh, I'm looking forward to that.

Tortoise: Well, Achilles, where are we?

Achilles (taking cognizance of his surroundings): We're in a little gondola, gliding down a canal! I want to get out. Mr.Gondolier, please let us out here.

(The gondolier pays no attention to this request.)

Tortoise: He doesn't speak English. If we want to get out here, we'd better just clamber out quickly before he

enters the sinister "Tunnel of Love", just ahead of us.

(Achilles, his face a little pale, scrambles out in a split second and then pulls his slower friend out.)

Achilles: I didn't like the sound of that place, somehow. I'm glad we got out here. Say, how do you know so much about this place, anyway? Have you been here before?

Tortoise: Many times, although I always came in from other Escher pictures. They're all connected behind the frames, you know. Once you're in one, you can get to any other one.

Achilles: Amazing! Were I not here, seeing these things with my own eyes, I'm not sure I'd believe you. (They wander out through a little arch.) Oh, look at those two cute lizards!

Tortoise: Cute? They aren't cute—it makes me shudder just to think of them! They are the vicious guardians of that magic copper lamp hanging from the ceiling over there. A mere touch of their tongues, and any mortal turns to a pickle.

Achilles: Dill, or sweet?

Tortoise: Dill.

Achilles: Oh, what a sour fate! But if the lamp has magical powers, I would like to try for it.

Tortoise: It's a foolhardy venture, my friend. I wouldn't risk it.

Achilles: I'm going to try just once.

(He stealthily approaches the lamp, making sure not to awaken the sleeping lad nearby. But suddenly, he slips on a strange shell-like indentation in the floor, and lunges out into space. Lurching crazily, he reaches for anything, and manages somehow to grab onto the lamp with one hand. Swinging wildly, with both lizards hissing and thrusting their tongues violently out at him, he is left dangling helplessly out in the middle of space.)

Achilles: He-e-e-elp!

(His cry attracts the attention of a woman who rushes downstairs and awakens the sleeping boy. He takes stock of the situation, and, with a kindly smile on his face, gestures to Achilles that all will be well. He shouts something in a strange guttural tongue to a pair of trumpeters high up in windows, and immediately, weird tones begin ringing out and making beats with each other. The sleepy young lad points at the lizards, and Achilles sees that the music is having a strong soporific effect on them. Soon, they are completely unconscious. Then the helpful lad shouts to two companions climbing up ladders. They both pull their ladders up and then extend them out into space just underneath the stranded Achilles, forming a sort of bridge. Their gestures make it clear that Achilles should hurry and climb on. But before he does so, Achilles carefully unlinks the top link of the chain holding the lamp, and detaches the lamp. Then he climbs onto the ladder-bridge and the three young lads pull him in to safety. Achilles throws his arms around them and hugs them gratefully.)

Achilles: Oh, Mr. T, how can I repay them?

Tortoise: I happen to know that these valiant lads just love coffee, and down in the town below, there's a place where they make an incomparable cup of espresso. Invite them for a cup of espresso!

Achilles: That would hit the spot.

(And so, by a rather comical series of gestures, smiles, and words, Achilles manages to convey his invitation to the young lads, and the party of five walks out and down a steep staircase descending into the town. They reach a charming small café, sit down outside, and order five espressos. As they sip their drinks, Achilles remembers he has the lamp with him.)

Achilles: I forgot, Mr. Tortoise—I've got this magic lamp with me! But—what's magic about it?

Tortoise: Oh, you know, just the usual—a genie.

Achilles: What? You mean a genie comes out when you rub it, and grants you wishes?

Tortoise: Right. What did you expect? Pennies from heaven?

Achilles: Well, this is fantastic! I can have any wish I want, eh? I've always wished this would happen to me ...

(And so Achilles gently rubs the large letter 'L' which is etched on the lamp's copper surface . . . Suddenly a huge puff of smoke appears, and in the forms of the smoke the five friends can make out a weird, ghostly figure towering above them.)

Genie: Hello, my friends—and thanks ever so much for rescuing my Lamp from the evil Lizard-Duo.

(And so saying, the Genie picks up the Lamp, and stuffs it into a pocket concealed among the folds of his long ghostly robe which swirls out of the Lamp.)

As a sign of gratitude for your heroic deed, I would like to offer you, on the part of my Lamp, the opportunity to have any three of your wishes realized.

Achilles: How stupefying! Don't you think so, Mr. T? Tortoise: I surely do. Go ahead, Achilles, take the first wish.

Achilles: Wow! But what should I wish? Oh, I know! It's what I thought of the first time I read the Arabian Nights (that collection of silly (and nested) tales)—I wish that I had a HUNDRED wishes, instead of just three! Pretty clever, eh, Mr. T? I bet YOU never would have thought of that trick. I always wondered why those dopey people in the stories never tried it themselves.

Tortoise: Maybe now you'll find out the answer.

Genie: I am sorry, Achilles, but I don't grant metawishes.

Achilles: I wish you'd tell me what a "meta-wish" is!

Genie: But THAT is a meta-meta-wish, Achilles—and I don't grant them, either.

Achilles: Whaaat? I don't follow you at all.

Tortoise: Why don't you rephrase your last request, Achilles?

Achilles: What do you mean? Why should I?

Tortoise: Well, you began by saying "I wish". Since you're just asking for information, why don't you just ask a question?

Achilles: All right, though I don't see why. Tell me, Mr. Genie—what is a meta-wish?

Genie: It is simply a wish about wishes. I am not allowed to grant meta-wishes. It is only within my purview to grant plain ordinary wishes, such as wishing for ten bottles of beer, to have Helen of Troy on a blanket, or to have an all-expenses-paid weekend for two at the Copacabana. You know—simple things like that. But meta-wishes I cannot grant. GOD won't permit me to.

Achilles: GOD? Who is GOD? And why won't he let you grant meta-wishes? That seems like such a puny thing compared to the others you mentioned.

Genie: Well, it's a complicated matter, you see. Why don't you just go ahead and make your three wishes? Or at least make one of them. I don't have all the time in the world, you know . . .

Achilles: Oh, I feel so rotten. I was REALLY HOPING to wish for a hundred wishes . . .

Genie: Gee, I hate to see anybody so disappointed as that. And besides, meta-wishes are my favorite kind of wish. Let me just see if there isn't anything I can do about this. This'll just take one moment—

(The Genie removes from the wispy folds of his robe an object which looks just like the copper Lamp he had put away, except that this one is made of silver; and where the previous one had 'L' etched on it, this one has 'ML' in smaller letters, so as to cover the same area.)

Achilles: And what is that?

Genie: This is my Meta-Lamp ...

(He rubs the Meta-Lamp, and a huge puff of smoke appears. In the billows of smoke, they can all make out a ghostly form towering above them.)

Meta-Genie: I am the Meta-Genie. You summoned me, O Genie? What is your wish?

Genie: I have a special wish to make of you, O Djinn, and of GOD. I wish for permission for temporary suspension of all type-restrictions on wishes, for the duration of one Typeless Wish. Could you please grant this wish for me?

Meta-Genie: I'll have to send it through Channels, of course. One half a moment, please.

(And, twice as quickly as the Genie did, this Meta-Genie removes from the wispy folds of her robe an object which looks just like the silver Meta-Lamp, except that it is made of gold; and where the previous one had 'ML' etched on it, this one has 'MML' in smaller letters, so as to cover the same area.)

Achilles (his voice an octave higher than before): And what is that?

Meta-Genie: This is my Meta-Meta-Lamp . . .

(She rubs the Meta-Meta-Lamp, and a huge puff of smoke appears. In the billows of smoke, they can all make out a ghostly form towering above them.)

Meta-Meta-Genie: I am the Meta-Meta-Genie. You summoned me, O Meta-Genie? What is your wish?

Meta-Genie: I have a special wish to make of you, O Djinn, and of GOD. I wish for permission for temporary suspension of all type-restrictions on wishes, for the duration of one Typeless Wish. Could you please grant this wish for me?

Meta-Meta-Genie: I'll have to send it through Channels, of course. One quarter of a moment, please.

(And, twice as quickly as the Meta-Genie did, this Meta-Meta-Genie removes from the folds of his robe an object which looks just like the gold Meta-Lamp, except that it is made of ...)

 $\{GOD\}$

(... swirls back into the Meta-Meta-Meta-Lamp, which the Meta-Meta-Genie then folds back into his robe, half as quickly as the Meta-Meta-Meta-Genie did.)

Your wish is granted, O Meta-Genie.

Meta-Genie: Thank you, O Djinn, and GOD.

(And the Meta-Meta-Genie, as all the higher ones before him, swirls back into the Meta-Meta-Lamp, which the Meta-Genie then folds back into her robe, half as quickly as the Meta-Meta-Genie did.)

Your wish is granted, O Genie. Genie: Thank you, O Djinn, and GOD.

(And the Meta-Genie, as all the higher ones before her,

swirls back into the Meta-Lamp, which the Genie then folds back into his robe, half as quickly as the Meta-Genie did.)

Your wish is granted, Achilles.

(And one precise moment has elapsed since he said "This will just take one moment.")

Achilles: Thank you, O Djinn, and GOD.

Genie: I am pleased to report, Achilles, that you may have exactly one (1) Typeless Wish—that is to say, a wish, or a meta-wish, or a meta-meta-wish, as many "meta"'s as you wish—even infinitely many (if you wish).

Achilles: Oh, thank you so very much, Genie. But my curiosity is provoked. Before I make my wish, would you mind telling me who—or what—GOD is?

Genie: Not at all. "GOD" is an acronym which stands for "GOD Over Djinn". The word "Djinn" is used to designate Genies, Meta-Genies, Meta-Meta-Genies, etc. It is a Typeless word.

Achilles: But—but—how can "GOD" be a word in its own acronym? That doesn't make any sense!

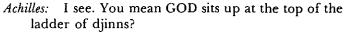
Genie: Oh, aren't you acquainted with recursive acronyms? I thought everybody knew about them. You see, "GOD" stands for "GOD Over Djinn"—which can be expanded as "GOD Over Djinn, Over Djinn"—and that can, in turn, be expanded to "GOD Over Djinn, Over Djinn, Over Djinn, Over Djinn"—which can, in its turn, be further expanded . . . You can go as far as you like.

Achilles: But I'll never finish!

Genie: Of course not. You can never totally expand GOD.

Achilles: Hmm... That's puzzling. What did you mean when you said to the Meta-Genie, "I have a special wish to make of you, O Djinn, and of GOD"?

Genie: I wanted not only to make a request of the Meta-Genie, but also of all the Djinns over her. The recursive acronym method accomplishes this quite naturally. You see, when the Meta-Genie received my request, she then had to pass it upwards to her GOD. So she forwarded a similar message to the Meta-Meta-Genie, who then did likewise to the Meta-Meta-Meta-Genie . . . Ascending the chain this way transmits the message to GOD.



Genie: No, no, no! There is nothing "at the top", for there is no top. That is why GOD is a recursive acronym. GOD is not some ultimate djinn; GOD is the tower of djinns above any given djinn.

Tortoise: It seems to me that each and every djinn would have a different concept of what GOD is, then, since to any djinn, GOD is the set of djinns above him or her, and no two djinns share that set.

Genie: You're absolutely right—and since I am the lowest djinn of all, my notion of GOD is the most exalted one. I pity the higher djinns, who fancy themselves somehow closer to GOD. What blasphemy!

Achilles: By gum, it must have taken genies to invent GOD.

Tortoise: Do you really believe all this stuff about GOD, Achilles?

Achilles: Why certainly, I do. Are you atheistic, Mr. T? Or are you agnostic?

Tortoise: I don't think I'm agnostic. Maybe I'm meta-agnostic.

Achilles: Whaaat? I don't follow you at all.

Tortoise: Let's see ... If I were meta-agnostic, I'd be confused over whether I'm agnostic or not—but I'm not quite sure if I feel THAT way; hence I must be meta-meta-agnostic (I guess). Oh, well. Tell me, Genie, does any djinn ever make a mistake, and garble up a message moving up or down the chain?

Genie: This does happen; it is the most common cause for Typeless Wishes not being granted. You see, the chances are infinitesimal that a garbling will occur at any PARTICULAR link in the chain—but when you put an infinite number of them in a row, it becomes virtually certain that a garbling will occur SOME-WHERE. In fact, strange as it seems, an infinite number of garblings usually occur, although they are very sparsely distributed in the chain.

Achilles: Then it seems a miracle that any Typeless Wish ever gets carried out.

Genie: Not really. Most garblings are inconsequential, and many garblings tend to cancel each other out. But occasionally—in fact, rather seldom—the nonfulfillment of a Typeless Wish can be traced back to a single unfortunate djinn's garbling. When this happens, the guilty djinn is forced to run an infinite

gauntlet, and get paddled on his or her rump, by GOD. It's good fun for the paddlers, and quite harmless for the paddlee. You might be amused by the sight.

Achilles: I would love to see that! But it only happens when a Typeless Wish goes ungranted?

Genie: That's right.

Achilles: Hmm . . . That gives me an idea for my wish.

Tortoise: Oh, really? What is it?

Achilles: I wish my wish would not be granted!

(At that moment, an event—or is "event" the word for it?—takes place which cannot be described, and hence no attempt will be made to describe it.)

Achilles: What on earth does that cryptic comment mean?

Tortoise: It refers to the Typeless Wish Achilles made.

Achilles: But he hadn't yet made it.

Tortoise: Yes, he had. He said, "I wish my wish would not be granted", and the Genie took THAT to be his wish.

(At that moment, some footsteps are heard coming down the hallway in their direction.)

Achilles: Oh, my! That sounds ominous.

(The footsteps stop; then they turn around and fade away.)

Tortoise: Whew!

Achilles: But does the story go on, or is that the end? Turn the page and let's see.

(The Tortoise turns the page of "Djinn and Tonic", where they find that the story goes on . . .)

Achilles: Hey! What happened? Where is my Genie? My lamp? My cup of espresso? What happened to our young friends from the Convex and Concave worlds? What are all those little lizards doing here?

Tortoise: I'm afraid our context got restored incorrectly, Achilles.

Achilles: What on earth does that cryptic comment mean?

Tortoise: I refer to the Typeless Wish you made.

Achilles: But I hadn't yet made it.

Tortoise: Yes, you had. You said, "I wish my wish would not be granted", and the Genie took THAT to be your wish.

Achilles: Oh, my! That sounds ominous.

Tortoise: It spells PARADOX. For that Typeless Wish to be

granted, it had to be denied—yet not to grant it would be to grant it.

Achilles: So what happened? Did the earth come to a standstill? Did the universe cave in?

Tortoise: No. The System crashed.

Achilles: What does that mean?

Tortoise: It means that you and I, Achilles, were suddenly and instantaneously transported to Tumbolia.

Achilles: To where?

Tortoise: Tumbolia: the land of dead hiccups and extinguished light bulbs. It's a sort of waiting room, where dormant software waits for its host hardware to come back up. No telling how long the System was down, and we were in Tumbolia. It could have been moments, hours, days—even years.

Achilles: I don't know what software is, and I don't know what hardware is. But I do know that I didn't get to make my wishes! I want my Genie back!

Tortoise: I'm sorry, Achilles—you blew it. You crashed the System, and you should thank your lucky stars that we're back at all. Things could have come out a lot worse. But I have no idea where we are.

Achilles: I recognize it now—we're inside another of Escher's pictures. This time it's Reptiles.

Tortoise: Aha! The System tried to save as much of our context as it could before it crashed, and it got as far as recording that it was an Escher picture with lizards before it went down. That's commendable.

Achilles: And look—isn't that our phial of poppingtonic over there on the table, next to the cycle of lizards?

Tortoise: It certainly is, Achilles. I must say, we are very lucky indeed. The System was very kind to us, in giving us back our popping-tonic—it's precious stuff!

Achilles: I'll say! Now we can pop back out of the Escher world, into my house.

Tortoise: There are a couple of books on the desk, next to the tonic. I wonder what they are. (He picks up the smaller one, which is open to a random page.) This looks like a moderately provocative book.

Achilles: Oh, really? What is its title?

Tortoise: Provocative Adventures of the Tortoise and Achilles Taking Place in Sundry Parts of the Globe. It sounds like an interesting book to read out of.

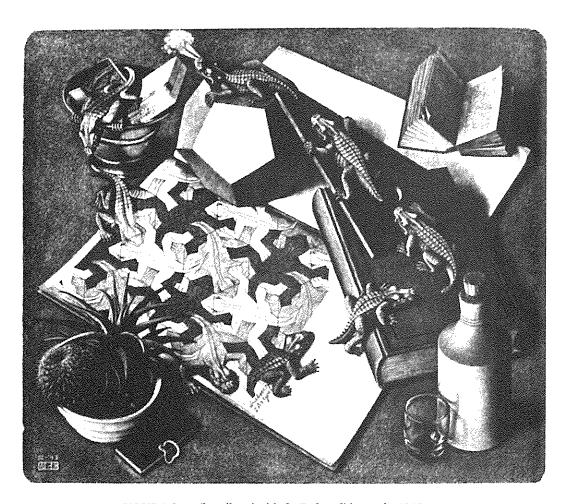


FIGURE 24. Reptiles, by M. C. Escher (lithograph, 1943).

Achilles: Well, YOU can read it if you want, but as for me, I'm not going to take any chances with that popping-tonic—one of the lizards might knock it off the table, so I'm going to get it right now!

(He dashes over to the table and reaches for the popping-tonic, but in his haste he somehow bumps the flash of tonic, and it tumbles off the desk and begins rolling.)

Oh, no! Mr. T—look! I accidentally knocked the tonic onto the floor, and it's rolling towards—towards—the stairwell! Quick—before it falls!

(The Tortoise, however, is completely wrapped up in the thin volume which he has in his hands.)

Tortoise (muttering): Eh? This story looks fascinating.

Achilles: Mr. T, Mr. T, help! Help catch the tonic-flask!

Tortoise: What's all the fuss about?

Achilles: The tonic-flask—I knocked it down from the desk, and now it's rolling and—

(At that instant it reaches the brink of the stairwell, and plummets over . . .)

Oh no! What can we do? Mr. Tortoise—aren't you alarmed? We're losing our tonic! It's just fallen down the stairwell! There's only one thing to do! We'll have to go down one story!

Tortoise: Go down one story? My pleasure. Won't you join me?

(He begins to read aloud, and Achilles, pulled in two directions at once, finally stays, taking the role of the Tortoise.)

Achilles: It's very dark here, Mr. T. I can't see a thing. Oof! I bumped into a wall. Watch out!

Tortoise: Here—I have a couple of walking sticks. Why don't you take one of them? You can hold it out in front of you so that you don't bang into things.

Achilles: Good idea. (He takes the stick.) Do you get the sense that this path is curving gently to the left as we walk?

Tortoise: Very slightly, yes.

Achilles: I wonder where we are. And whether we'll ever see the light of day again. I wish I'd never listened to you, when you suggested I swallow some of that "DRINK ME" stuff.

Tortoise: I assure you, it's quite harmless. I've done it scads of times, and not a once have I ever regretted it. Relax and enjoy being small.

Achilles: Being small? What is it you've done to me, Mr. T?

Tortoise: Now don't go blaming me. You did it of your own free will.

Achilles: Have you made me shrink? So that this labyrinth we're in is actually some teeny thing that someone could STEP on?

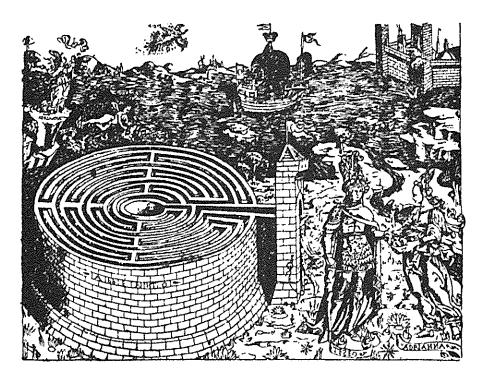


FIGURE 25. Cretan Labyrinth (Italian engraving; School of Finiguerra). [From W. H. Matthews, Mazes and Labyrinths: Their History and Development (New York: Dover Publications, 1970).]

Tortoise: Labyrinth? Labyrinth? Could it be? Are we in the notorious Little Harmonic Labyrinth of the dreaded Majotaur?

Achilles: Yiikes! What is that?

Tortoise: They say—although I personally never believed it myself—that an Evil Majotaur has created a tiny labyrinth and sits in a pit in the middle of it, waiting for innocent victims to get lost in its fearsome complexity. Then, when they wander lost and dazed into the center, he laughs and laughs at them—so hard, that he laughs them to death!

Achilles: Oh, no!

Tortoise: But it's only a myth. Courage, Achil-

les.

(And the dauntless pair trudge on.)

Achilles: Feel these walls. They're like corrugated tin sheets, or something. But the corrugations have different sizes.

(To emphasize his point, he sticks out his walking stick against the wall surface as he walks. As the stick bounces back and forth against the corrugations, strange noises echo up and down the long curved corridor they are in.)

Tortoise (alarmed): What was THAT?

Achilles: Oh, just me, rubbing my walking stick against the wall.

Tortoise: Whew! I thought for a moment it was the bellowing of the ferocious Majotaur!

Achilles: I thought you said it was all a myth.

Tortoise: Of course it is. Nothing to be afraid of.

(Achilles puts his walking stick back against the wall, and continues walking. As he does so, some musical sounds are heard, coming from the point where his stick is scraping the wall.)

Tortoise: Uh-oh. I have a bad feeling, Achilles. That Labyrinth may not be a myth, after all.

Achilles: Wait a minute. What makes you change your mind all of a sudden?

Tortoise: Do you hear that music?

(To hear more clearly, Achilles lowers the stick, and the strains of melody cease.)

Hey! Put that back! I want to hear the end of this piece!

(Confused, Achilles obeys, and the music resumes.)

Thank you. Now as I was about to say, I have just figured out where we are.

Achilles: Really? Where are we?

Tortoise: We are walking down a spiral groove of a record in its jacket. Your stick scraping against the strange shapes in the wall acts like a needle running down the groove, allowing us to hear the music.

Achilles: Oh, no, oh, no . . .

Tortoise: What? Aren't you overjoyed? Have you ever had the chance to be in such intimate contact with music before?

Achilles: How am I ever going to win footraces against full-sized people when I am smaller than a flea, Mr. Tortoise?

Tortoise: Oh, is that all that's bothering you? That's nothing to fret about, Achilles.

Achilles: The way you talk, I get the impression that you never worry at all.

Tortoise: I don't know. But one thing for certain is that I don't worry about being small.

Especially not when faced with the awful danger of the dreaded Majotaur!

Achilles: Horrors! Are you telling me-

Tortoise: I'm afraid so, Achilles. The music gave it away.

Achilles: How could it do that?

Tortoise: Very simple. When I heard the melody B-A-C-H in the top voice, I immediately realized that the grooves that we're walking through could only be the Little Harmonic Labyrinth, one of Bach's lesser known organ pieces. It is so named because of its dizzyingly frequent modulations.

Achilles: Wh-what are they?

Tortoise: Well, you know that most musical pieces are written in a key, or tonality, such as C major, which is the key of this one.

Achilles: I had heard the term before. Doesn't that mean that C is the note you want to end on?

Tortoise: Yes, C acts like a home base, in a way. Actually, the usual word is "tonic".

Achilles: Does one then stray away from the tonic with the aim of eventually returning?

Tortoise: That's right. As the piece develops, ambiguous chords and melodies are used, which lead away from the tonic. Little by little, tension builds up—you feel an increasing desire to return home, to hear the tonic.

Achilles: Is that why, at the end of a piece, I always feel so satisfied, as if I had been waiting my whole life to hear the tonic?

Tortoise: Exactly. The composer has used his knowledge of harmonic progressions to

manipulate your emotions, and to build up hopes in you to hear that tonic.

Achilles: But you were going to tell me about modulations.

Tortoise: Oh, yes. One very important thing a composer can do is to "modulate" partway through a piece, which means that he sets up a temporary goal other than resolution into the tonic.

Achilles: I see ... I think. Do you mean that some sequence of chords shifts the harmonic tension somehow so that I actually desire to resolve in a new key?

Tortoise: Right. This makes the situation more complex, for although in the short term you want to resolve in the new key, all the while at the back of your mind you retain the longing to hit that original goal—in this case, C major. And when the subsidiary goal is reached, there is—

Achilles (suddenly gesturing enthusiastically): Oh, listen to the gorgeous upward-swooping chords which mark the end of this Little Harmonic Labyrinth!

Tortoise: No, Achilles, this isn't the end. It's merely—

Achilles: Sure it is! Wow! What a powerful, strong ending! What a sense of relief! That's some resolution! Gee!

(And sure enough, at that moment the music stops, as they emerge into an open area with no walls.)

You see, it IS over. What did I tell you? Tortoise: Something is very wrong. This record is a disgrace to the world of music.

Achilles: What do you mean?

Tortoise: It was exactly what I was telling you about. Here Bach had modulated from C into G, setting up a secondary goal of hearing G. This means that you experience two tensions at once—waiting for resolution into G, but also keeping in mind that ultimate desire—to resolve triumphantly into C Major.

Achilles: Why should you have to keep any-

thing in mind when listening to a piece of music? Is music only an intellectual exercise?

Tortoise: No, of course not. Some music is highly intellectual, but most music is not. And most of the time your ear or brain does the "calculation" for you, and lets your emotions know what they want to hear. You don't have to think about it consciously. But in this piece, Bach was playing tricks, hoping to lead you astray. And in your case, Achilles, he succeeded.

Achilles: Are you telling me that I responded to a resolution in a subsidiary key?

Tortoise: That's right.

with you!

Achilles: It still sounded like an ending to me.

Tortoise: Bach intentionally made it sound that way. You just fell into his trap. It was deliberately contrived to sound like an ending, but if you follow the harmonic progression carefully, you will see that it is in the wrong key. Apparently not just you but also this miserable record company fell for the same trick—and they truncated the piece early!

Achilles: What a dirty trick Bach played on me!

Tortoise: That is his whole game—to make you lose your way in his Labyrinth! The Evil Majotaur is in cahoots with Bach, you see. And if you don't watch out, he will now laugh you to death—and perhaps me along

Achilles: Oh, let us hurry up and get out of here! Quick! Let's run backwards in the grooves, and escape on the outside of the record before the Evil Majotaur finds us!

Tortoise: Heavens, no! My sensibility is far too delicate to handle the bizarre chord progressions which occur when time is reversed.

Achilles: Oh, Mr. T, how will we ever get out of here, if we can't just retrace our steps?

Tortoise: That's a very good question.

(A little desperately, Achilles starts running about aimlessly in the dark. Suddenly there is a slight gasp, and then a "thud".)

Achilles-are you all right?

Achilles: Just a bit shaken up but otherwise fine. I fell into some big hole.

Tortoise: You've fallen into the pit of the Evil Majotaur! Here, I'll come help you out. We've got to move fast!

Achilles: Careful, Mr. T—I don't want YOU to fall in here, too . . .

Tortoise: Don't fret, Achilles. Everything will be all—

(Suddenly, there is a slight gasp, and then a "thud".)

Achilles: Mr. T—you fell in, too! Are you all right?

Tortoise: Only my pride is hurt—otherwise I'm fine.

Achilles: Now we're in a pretty pickle, aren't we?

(Suddenly, a giant, booming laugh is heard, alarmingly close to them.)

Tortoise: Watch out, Achilles! This is no laughing matter.

Majotaur: Hee hee! Ho ho! Haw haw! Achilles: I'm starting to feel weak, Mr. T...

Tortoise: Try to pay no attention to his laugh, Achilles. That's your only hope.

Achilles: I'll do my best. If only my stomach weren't empty!

Tortoise: Say, am I smelling things, or is there a bowl of hot buttered popcorn around here?

Achilles: I smell it, too. Where is it coming from?

Tortoise: Over here, I think. Oh! I just ran into a big bowl of the stuff. Yes, indeed—it seems to be a bowl of popcorn!

Achilles: Oh, boy—popcorn! I'm going to munch my head off!

Tortoise: Let's just hope it isn't pushcorn! Pushcorn and popcorn are so extraordinarily difficult to tell apart.

Achilles: What's this about Pushkin?

Tortoise: I didn't say a thing. You must be hearing things.

Achilles: Go-golly! I hope not. Well, let's dig in!

(And the two friends begin munching the popcorn (or pushcorn?)—and all at once—POP! I guess it was popcorn, after all.)

Tortoise: What an amusing story. Did you enjoy it?

Achilles: Mildly. Only I wonder whether they ever got out of that Evil Majotaur's pit or not. Poor Achilles—he wanted to be full-sized again.

Tortoise: Don't worry—they're out, and he is full-sized again. That's what the "POP" was all about.

Achilles: Oh, I couldn't tell. Well, now I REALLY want to find that bottle of tonic. For some reason, my lips are burning. And nothing would taste better than a drink of popping-tonic.

Tortoise: That stuff is renowned for its thirst quenching powers. Why, in some places people very nearly go crazy over it. At the turn of the century in Vienna, the Schönberg food factory stopped making tonic, and started making cereal instead. You can't imagine the uproar that caused.

Achilles: I have an inkling. But let's go look for the tonic. Hey—just a moment. Those lizards on the desk—do you see anything funny about them?

Tortoise: Umm... not particularly. What do you see of such great interest?

Achilles: Don't you see it? They're emerging from that flat picture without drinking any popping-tonic! How are they able to do that?

Tortoise: Oh, didn't I tell you? You can get out of a picture by moving perpendicularly to its plane, if you have no popping-tonic. The little lizards have learned to climb UP when they want to get out of the two-dimensional sketchbook world.

Achilles: Could we do the same thing to get out of this Escher picture we're in?

Tortoise: Of course! We just need to go UP one story. Do you want to try it?

Achilles: Anything to get back to my house! I'm tired of all these provocative adventures.

Tortoise: Follow me, then, up this way.

(And they go up one story.)

Achilles: It's good to be back. But something seems wrong. This isn't my house! This is YOUR house, Mr. Tortoise.

Tortoise: Well, so it is-and am I glad for that! I wasn't looking

forward one whit to the long walk back from your house. I am bushed, and doubt if I could have made it.

Achilles: I don't mind walking home, so I guess it's lucky we ended up here, after all.

Tortoise: I'll say! This certainly is a piece of Good Fortune!