The Multicultural Math Classroom

Bringing in the World

Claudia Zaslavsky

HEINEMANN
Portsmouth, NH

Chapter 9

Geometry, Measurement, & Symmetry in Art

Mathematical Topics
Geometry of shapes, measurement of angles and polygons, scale drawing, similarity, symmetry, repeated patterns, tessellations.

Cultural Connections
Africa: Ghana (Asante), Zaire (Kuba); Asia: China, Japan, Islam; North America: Yup'ik (southwest Alaska Eskimo), United States in general.

Linked Subject
Art: tangrams, paper folding, quilting, border patterns, block printing, tessellations.

In many societies art is widely diffused throughout the culture and the life of the individual. Art is not just something to hang on the wall or to display in a museum. Articles of everyday use, such as pottery, basketry, and textiles, are decorated in patterns that often have symbolic meaning. Indeed, archaeologists work hand in hand with mathematicians to identify ancient societies by analyzing patterns in decorated pottery and other surviving objects. A beautifully carved gameboard may be passed down from generation to generation. Art is also an integral part of the religious life of the community, as exemplified by masks, ceremonial dress, religious objects, and architectural elements.

Bringing math to life through the medium of art gives students the kinds of physical experiences that are essential for the development of spatial thinking. Geometric concepts have real meaning when they are presented in this concrete form. Art activities are open-ended; each student can take a project as far and in whatever direction he or she pleases. Beyond the mathematical benefits, art is a source of joy to young people. They can learn math, be creative, and have fun all at the same time.

Women have an important role in creating art. Sometimes the products of so-called women's work are assigned a low status by being referred to as crafts rather than art; this is true also of the works of non-Western societies. In her Maths in Work Project, British mathematician Mary Harris revealed the mathematics inherent in traditional women's work with textiles. One display in her traveling exhibit *Common Threads* compares the mathematical aspects of a design for a right-angled cylindrical pipe in a chemical factory with the mathematics of knitting the heel of a sock, and raises the question, Why is the industrial problem considered real mathematics, while the knitting problem is not taken seriously as valid mathematics?

Through art as the medium, the school can involve the families and communities of the students. Children may learn from their parents or grandparents some of the techniques that are getting lost in our machine-made culture. Invite local artists to visit the classroom and demonstrate their expertise. Families may want to exhibit art objects from other lands and the distant past.

Art is a rich field for the exploration of many concepts in geometry and measurement—size, shape, angle measurement, similarity, symmetry, and repeated patterns, including tessellations. Any of the activities suggested below involves more than one of these concepts. I will discuss several types of activities as examples of the possibilities in the mathematics-art connection. You may want to link such activities to lessons in art or to the societies discussed in social studies classes.

POLYGONS

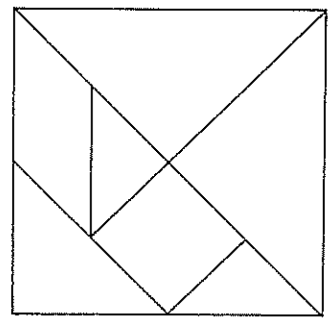
Discussion and Activities

Observe the environment

Ask students to point out various polygons that they see in their environment and identify them by name, if possible. Which polygons are regular—all sides of equal length and all angles of equal measurement?

Tangrams

Tangrams originated in China and became popular in other parts of the world in the nineteenth century. Tangram pieces are seven polygons cut from a square so that the measures of their sides are in specific relationships.



Shape, area, and perimeter. Challenge students to arrange the *tans* to form pleasing designs or to rearrange the pieces to form other polygons, including the original square. Do students understand that all the figures they create have the same area (provided they use all seven tans), no matter how the shape changes? They might also note that the perimeters vary while the area remains the same.

Young children will enjoy *Grandfather Tang's Story* (Tompert 1990), about two animals who escape being caught by transforming themselves into different shapes. Various combinations of tangrams illustrate the shapes of the animals.

?APER FOLDING

3ackground

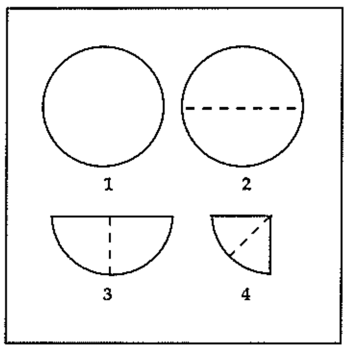
Who has not done paper folding and cutting to make Valentine hearts, Christmas trees, Thanksgiving pumpkins, and elaborate six-pointed snowflakes? Here we will discuss two types of Japanese paper-folding activities—*mon-kiri* and *origami.*

)iscussion and Activities

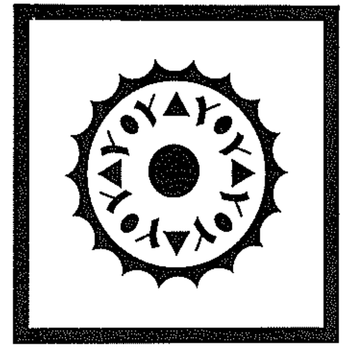
Mon-Kiri

The practice of cutting designs in paper has a long history in both China and Japan and requires a great deal of skill. Samurai warriors decorated their armor with these designs. The simple design in this activity is similar to a snowflake, except that it will have four-fold rather than six-fold symmetry. Dr. Beverly Ferrucci, an instructor at Keene State College in New Hampshire, has her students follow this procedure:

1. Cut a six-inch circle of white paper. Fold it four times so that it has eight layers.



2. Draw and cut a design. All lines must touch the outer edges and no lines may touch.
3. Open the design and glue it onto a six-inch square of black paper. You may want to trim the black paper before gluing it onto a seven-inch red square. Glue the red paper to an eight-inch black square.



Beverly poses the following questions:

1. What types of angles do you see in each step of the design?
2. How many layers of paper do you have after making each fold?
3. How do changes in your original cuts affect your design when it is open?

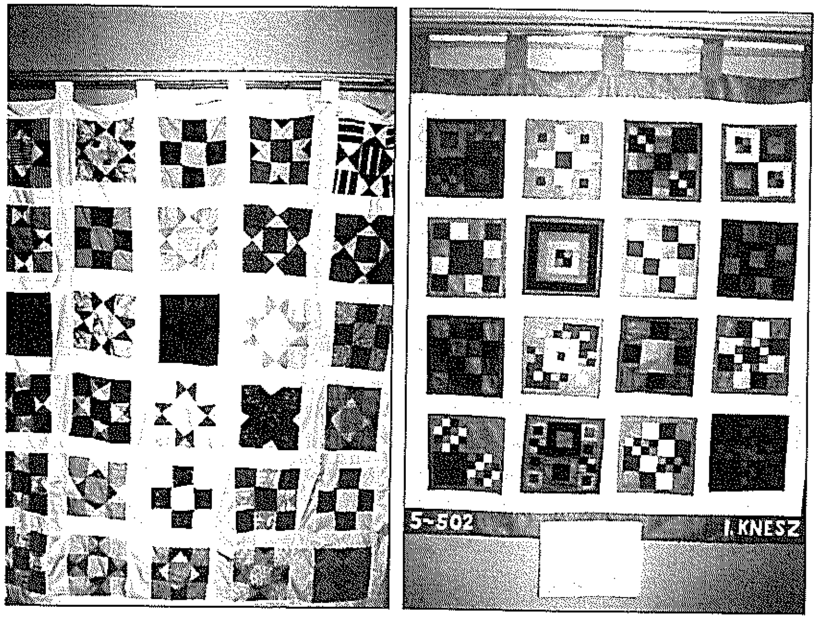
Origami

Introduce this ancient Japanese art through the book *Sadako and the Thousand Cranes* (Coerr 1977), a true story about a victim of radiation poisoning as a result of the bombing of Hiroshima in World War II. The belief was that she would live if she and her friends folded a thousand cranes. Sadly, she died before achieving this goal. Specific instructions for making origami figures are available in *The ABCs of Origami* (Sarasas 1964) and other references.

QUILT DESIGNS

Background

As I walked through the lobby of a local elementary school, two beautiful hangings, truly works of art, caught my attention. One was a patchwork quilt top of thirty squares in a five-by-six array. Geometrically the squares were identical, based on the combination of small squares and right triangles that form the Ohio Star motif. For a varied and



■ FIGURES 9–1A & B Patchwork and appliqué hangings by students of P.S. 189, Manhattan. Photographs by Sam Zaslavsky

pleasing effect, each square was composed of a different combination of patterned cloth, quite a feat for a fourth-grade class. The teacher explained that she had been an art major, the class was studying the colonial period, and she wanted them to get a feeling for life in those times. She spoke of art and social studies, but not of mathematics! When I asked whether the boys had objected to sewing, she said: "I told them that tailors sew. No problem whatsoever."

Next to the patchwork quilt was a fifth-grade art project consisting of sixteen felt squares to which had been sewed smaller felt squares of various sizes and colors. Each large square of the four-by-four array was unique. What an impressive way to use the square as a motif!

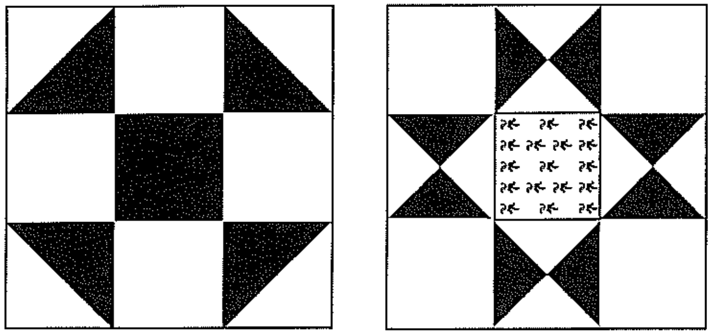
Quilting was introduced to the American colonies by settlers from England and the Netherlands and by Africans brought to the continent

as slaves. For women who were restricted in their activities, the quilting bee provided an acceptable social gathering. Women would gather to make quilts for special occasions—a wedding of a son or daughter, the arrival of a new minister, the departure of a family for the West. Quilting was often a woman's sole means of artistic expression. One woman wrote: "It took me near twenty-five years to make that quilt. My whole life is in that quilt." Some African American women were able to purchase their freedom from slavery with their beautiful quilts, often based on African patterns.

Both the designs and the quilts have been handed down from generation to generation. Some examples became part of museum collections, a celebration of women's contributions to the arts over the years. Quilting is becoming ever more popular as a hobby, while professional artists are testing their designs on the computer before applying them to cloth.

In her inspiring book *Garbage Pizza, Patchwork Quilts, and Math Magic* (1993), Susan Ohanian tells of an eastern Kentucky school district that was looking for a way to involve parents in their children's math learning. Many of the parents had not finished high school, and the prospect of doing math was intimidating. A paper quilt project, introduced through the Parent Art Council, was the solution. As the school coordinator commented, "It is easier to recruit parents for an art project than for a math project. We designed art projects that would reinforce math concepts" (185). Many parents were already knowledgeable about quilts and were eager to become involved. Ohanian writes, "Parents who have always considered themselves 'bad at math' see that they had very definite skills to share with the children, skills of academic value" (186).

The square is the basic shape for the patchwork quilt. The designs may vary from one square to the other, or the entire quilt may be composed of squares having identical designs by varied colors or types of fabric, like the one the fourth graders displayed in the school lobby. Because the Shoo Fly and Ohio Star motifs are among the simplest, I use them as a basis for discussion, but there is no need to limit yourself to these patterns. Teachers are often amazed by the complexity of their students' creations.



■ FIGURE 9–2 Quilt blocks: Shoo Fly and Ohio Star

Discussion and Activities

Observe quilt patterns

Young children will enjoy reading a story about quilts and seeing the pictures. Adult books with illustrations of the great variety of quilt blocks and photographs of actual quilts are intriguing to all students. Ohanian's book includes illustrations of many simple quilt blocks and paper quilts made by primary school children.

Quilt display

Students or their parents may have treasured quilts to display to the class. Perhaps a local quilt maker can be persuaded to visit and discuss his or her work.

Mathematical analysis of a quilt block

Ask the students to choose one quilt block. Let's assume that they have chosen either the Shoo Fly or Ohio Star pattern. Give the students copies of the pattern and ask them to write about it. They may note any of the following features, depending on the mathematical topics they have studied:

1. *Shapes.* Name the geometric shapes. How many of each? What are their colors? Describe or draw diagrams to show how you can put together two or more shapes to make a different shape or a larger shape of the same kind.

2. *Measurement.* Describe the relationships among the lengths of the sides of the different shapes. State the angle measurements of each shape.

3. *Symmetry.* Students may have questions about the symmetry of the designs in handmade quilts. Accustomed to the near perfection of design in factory-made patterns, they may consider the flaws in handcrafted quilts to be violations of symmetry. Encourage them to consider the intentions of the artist and to overlook insignificant or unavoidable deviations from perfection, an ideal that can never be achieved in the real world. Some kinds of symmetry include:

   a. *Line (or fold) symmetry.* Analyze the three-by-three square quilt block for line symmetry. Can you fold the block in half so that one half of the design matches the other half? An alternative is the mirror test. If you placed a mirror upright on the fold line, would the reflection in the mirror look the same as the half of the design behind the mirror? How many different fold lines are there? Make sketches, showing the fold lines as dotted lines. [Four: one horizontal, one vertical, and two diagonal. The square has *line symmetry of order four.*]

   b. *Rotational symmetry.* Analyze the three-by-three square quilt block for turn symmetry. Rotate the block slowly until the design looks the same as it did in the original position. In how many different positions does the design look the same, including the original position? Through what angle does the block turn each time? [Four; 90°. The square has *rotational symmetry of order four.*] A helpful (but not always feasible) test for turn symmetry is to trace the design on a sheet of thin paper and label one corner of both squares. Place a pin, pencil point, or fingertip on the center of both squares. Then rotate one square slowly until the designs on the two squares match. Continue rotating until the free square is back in its original position.

   c. *Color symmetry.* A design may have symmetry with respect to its shape but not its color. (To take an unlikely but easily explained example, suppose that the left half of the quilt block is done in shades of blue and the right half in shades of yellow.) Depending on students' age and mathematical background, encourage them to analyze both shape and color.

Sometimes a design is symmetrical except for one small element. Students should note such exceptions.

4. *Similarity.* Suppose you want to make a quilt block that has a side twice as long as the side of the model square. How many times as much paper or fabric will you need? Suppose it is three times as long? Can you make a general statement about similar squares? [Four; nine; if the quilt block is n times as long as the model, it will require n^2 as much material.]

Construct a quilt block
Have each student put together one three-by-three square quilt block of construction paper. Give each student nine identical squares of one color and nine of another color. They can cut some of the squares into triangles and put together quilt blocks of their own design.

Make a quilt
Encourage each group of students, or the class as a whole, to plan and construct a large quilt of construction paper, each student contributing one or more squares. Perhaps they are sufficiently ambitious to sew a fabric patchwork quilt top. They should start with a labeled scale drawing of the quilt block. Ask them to describe in writing, with illustrations, the reason they selected a particular design, how the design was constructed, colors they used, the geometric concepts, difficulties they encountered in carrying out the project, and how they felt about doing the project.

Computers
Students with some computer graphics experience and adequate facilities can design quilt squares on the computer. A good project would be for these students to write and illustrate a booklet about quilt design on the computer.

Exhibit
Arrange a quilt exhibit for the corridors of your school. Call in parents and the press to tell the community about it.

Extension to other shapes
Encourage students to analyze and write about symmetrical designs in various cultures, as they did with the quilt patterns. How is the design used, and what is its history and significance? The ankh, the peace symbol, the five- or six-pointed star, and the Chinese yin/yang all have distinctive geometric form and symmetry, as well as symbolic significance. The flags of many countries are a fruitful source of symbols.

REPEATED PATTERNS IN BORDERS

Yup'ik Border Patterns: Background

The Yup'ik (Eskimo) live along the southwestern coast of Alaska. The borders of Yup'ik parkas provide a fine example of traditional repeated patterns. During the first several weeks of school, Yup'ik teacher Esther Ilutsik spends a few minutes a week informally introducing her Yup'ik primary school students to the names and meaning of the different patterns. After her students have become familiar with the patterns, she has the children bring in (from home or from magazines) photos of parkas with border patterns. The children make photocopies of their pictures and combine them in a poster that identifies the appropriate name of each pattern; alternatively, they may draw their favorite patterns on a poster. As the children, working in pairs, learn to reproduce and color the patterns, Esther comments on their work and emphasizes the traditional Yup'ik pattern names, words meaning mountains, window, sled runners, and "put little things together." Drawing these patterns generates a great deal of enthusiasm, and all children eagerly join in the activity.

Esther continues: "The next few lessons are on reproducing patterns by name and having the children take a lead in this activity. For example, one child will ask the other children to create a 'braid' pattern. This child then checks work and the one who completes it first and correctly gets to be the next leader." She then combines two aspects of Yup'ik culture by having the children "try to put the patterns to drum beats. The other children try to find the correct pattern. The child then has to explain his 'rule.' "

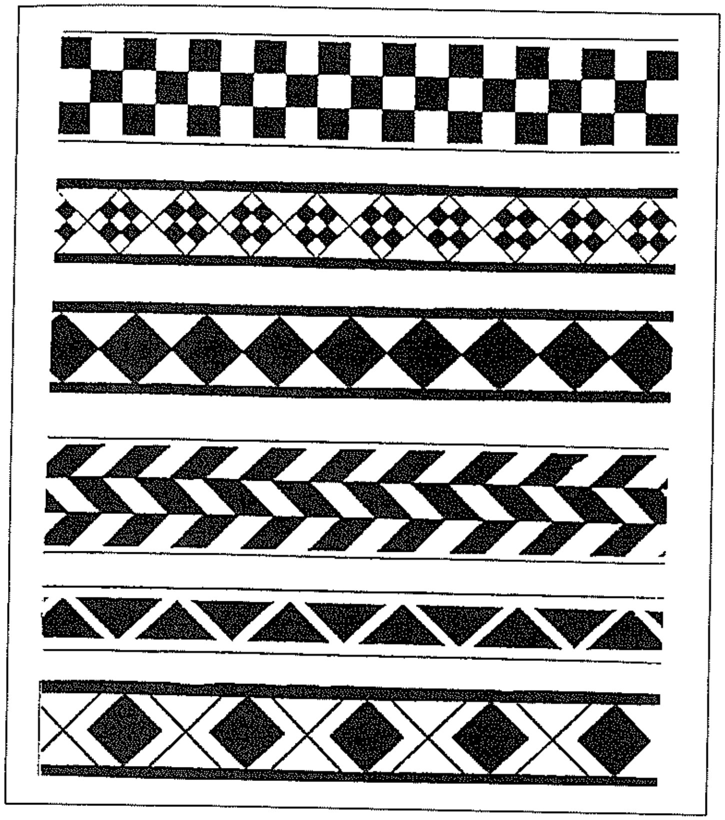


FIGURE 9–3 Traditional Yup'ik border patterns

Finally, after these lively activities, Esther discusses the geometric terms for the patterns—square, triangle, diamond, and so on—as well as the appropriate vocabulary for the operations—translations (slides), rotations (turns), reflections (flips), and magnification (stretching or enlarging).

I was invited to Alaska to attend a conference arranged by University of Alaska instructors for teachers and teacher aides in predominantly Yup'ik schools, as well as Yup'ik elders, held at the end

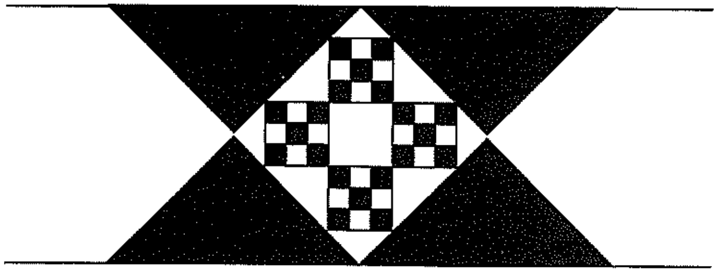
of the school year to plan curriculum for the following year. The topic under discussion one afternoon was patterns and symmetry. All the participants were women; the men were off in another room discussing the traditional Yup'ik calendar. Each person received an envelope of precut posterboard squares and isosceles right triangles, two sizes of each. After the participants had used the materials to form border patterns, Dr. Claudette Bradley, a University of Alaska instructor, reviewed the concepts of geometric transformations—symmetry, translation or slide, reflection, rotation, and magnification or stretching.

The next task was to assign proper Yup'ik terminology to each of these concepts, as bilingual teachers or aides translated for the elders who did not speak English. After much discussion in both Yup'ik and English, one of the teachers wrote the terms in both languages on the chalkboard. They translated *symmetry* into the Yup'ik words for *same measure*. Because of differences in the dialects of Yup'ik spoken in different regions of western Alaska, some terms were translated into two alternative forms. One translation of *reflection* meant *mirror*, while another borrowed from the terminology of sewing. A participant commented that in her region the vocabulary was slightly different and she would have to consult the elders when she went home.

One teacher displayed two parkas that her mother had made many years before. Both her mother and her aunt were among the elders at the conference, and you can imagine their pride in having this work used as a basis for mathematics lessons. Borders on parkas became the theme for the next activity. The participants worked in groups to design lessons based on the use of the manipulatives and the vocabulary of transformations. Each teacher or aide explained her lesson, both the mathematical and the cultural aspects, using the overhead projector to enlarge the patterns. Esther Ilutsik discussed the activities that she had already carried out the previous year, which I described above.

Kathryn Schubeck, Esther's fellow teacher at Aleknagik Elementary School, typed her lesson plan on the computer, drew the necessary illustrations based on the border of one of the models, and distributed copies to all the participants. In her lessons she covered not only the concepts that the group had discussed, but also area and the relationship among the three sides of a right triangle, all based on the repeating pattern in the parka border.

Before going on to patterns in dance fans, some of the participants



■ **FIGURE 9–4 Traditional Yup'ik parka border pattern**

played the following game. Two people sat back to back. One person made a pattern, which she described to her colleague in the Yup'ik language. That person then made the pattern based on her interpretation of the description. When they compared the outcomes, they analyzed their mistakes. Did they result from an insufficient or misleading description or from a failure to follow directions? Students will have a lot of fun playing that game.

athematical Analysis of Repeated Border Patterns

In a border pattern, a motif is repeated along one direction. The best way to analyze the mathematical operations is to start with a rectangular stencil into which a motif has been cut and a set of "tracks" along which the stencil will move. The operations are those we have already discussed—slide, half-turn, and flip, as the children like to say. In more mathematical language, we'll be working with operations called *translations* (slides), *rotations of 180°* (half-turns), and *reflections* in either a *horizontal* or *vertical axis* (flips). Another mathematical term is *glide reflection*, which consists of a horizontal flip and a slide in one operation. By repeating one operation or a combination of operations, we find that there are exactly seven different ways to repeat a pattern along one direction.

Start with a track of two parallel lines subdivided into rectangles to match the size and shape of the rectangular stencil. Then follow the directions for the first five types. The last two types require a double track to accommodate horizontal flips. The sequence of motions is

indicated by arrows and by the numbers next to each rectangle. See pages 154 and 155.

Discussion and Activities

Border patterns with stamps

Stamps can be made from such materials as sponges, potato halves, and foam pads with self-adhesive backing so that they can be attached to blocks of wood. Suggest that students use their stamps to create borders for place mats, book covers, etc. First they should draw a track. They might slide the stamp from one position to the next (Type 1), or give it a half-turn every time the design is repeated (Type 2).

Border patterns with stencils

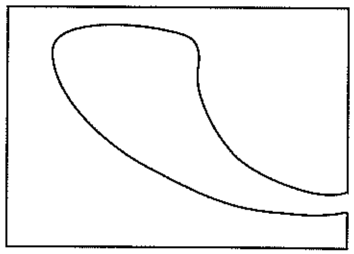
Students will find that they cannot use a stamp to produce reflections of the motif. They will need a different technique. One way is to cut out a design and trace it. An alternative is to make a stencil by cutting a design into a rectangle of heavy paper or plastic. The design should not have horizontal or vertical symmetry, because that might limit the variety of patterns. (Can students prove this statement?) Older students might study the seven types and select one or two types as a basis for their own creations.

Computers

Encourage students with access to computers and with the necessary know-how to develop a computer program that can generate repeated patterns for a border.

Observation in the environment

Once students have created their own repeated patterns, they will become aware of repeated patterns in their environment, in books and magazines. Encourage them to bring in examples and analyze the types of symmetry. Can they analyze the examples in this chapter? Look for the smallest region that might be a motif, a region that has no horizontal or vertical symmetry. Then compare the pattern with the seven types on page 155.



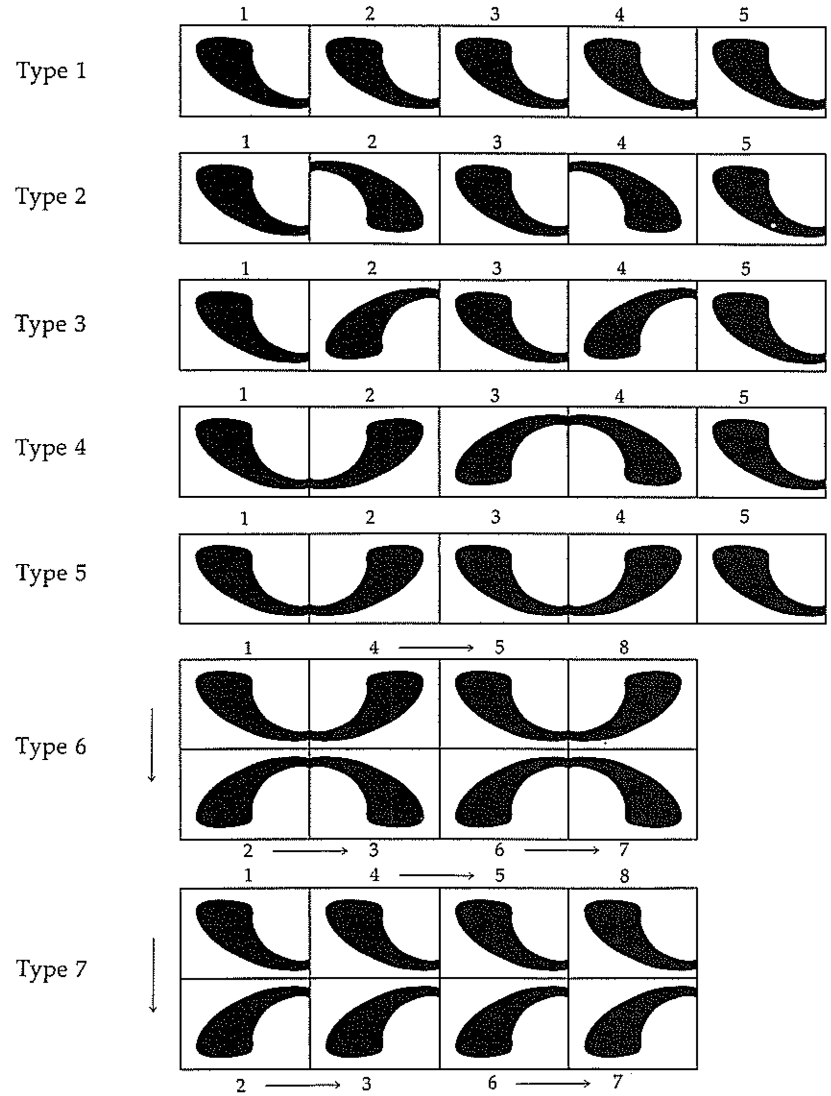
Code: H: flip about a horizontal axis
V: flip about a vertical axis

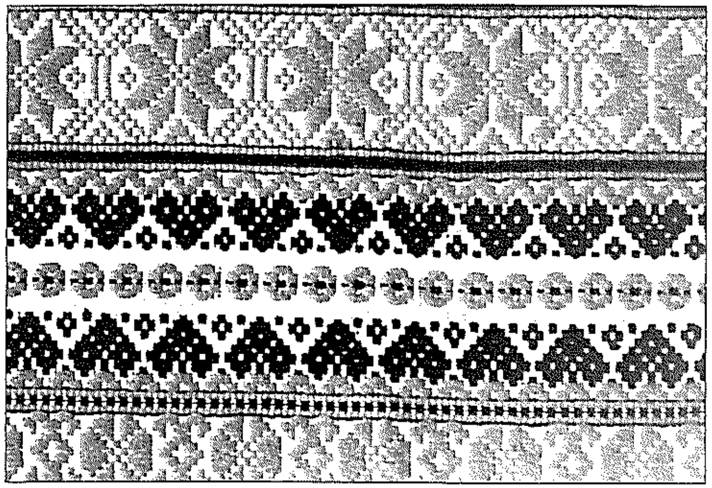
Type	Motion of stencil from one frame to the next			
	1 to 2	2 to 3	3 to 4	4 to 5
1	slide	repeat	repeat	repeat
2	half-turn	repeat	repeat	repeat
3	H-flip and slide	repeat	repeat	repeat
4	V-flip	half-turn	V-flip	half-turn
5	V-flip	repeat	repeat	repeat
6	H-flip	V-flip	H-flip	V-flip
7	H-flip	slide	H-flip	slide

REPEATING PATTERNS IN THE PLANE

Block Prints—*Adinkra*

Young children will enjoy making block prints in a free style before they do more formal work. *Adinkra* cloth is worn by the Asante (also spelled Ashanti) people of Ghana. The cloth consists of a number of rectangles sewed together, with colorful borders separating the pieces. Each rectangle is stamped with rows of a different motif, each with its own symbolism. A motif is cut into a piece of a calabash attached to a handle, which is then dipped into ink or paint.





■ FIGURE 9–5 Repeated patterns in Hungarian embroidery. Photograph by Sam Zaslavsky

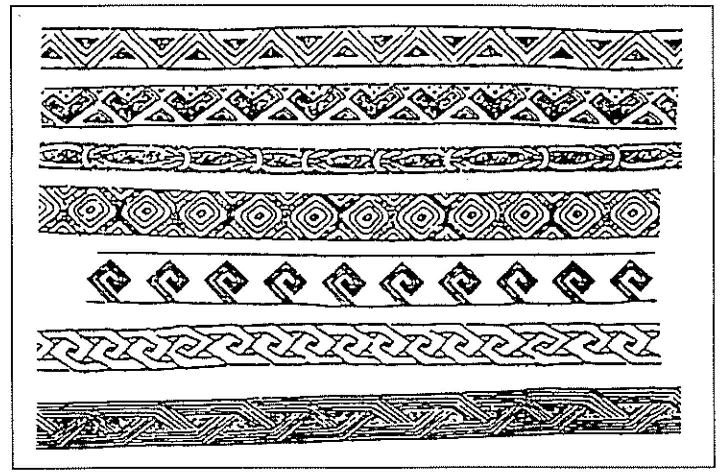
Activities

Suggest that students or groups of students design their own motifs and print *adinkra*-like rectangles on paper or cloth. Paste or sew colored strips to connect the rectangles. Students should research the significance of *adinkra* and write about their art production from both the mathematical and cultural point of view.

Tessellations

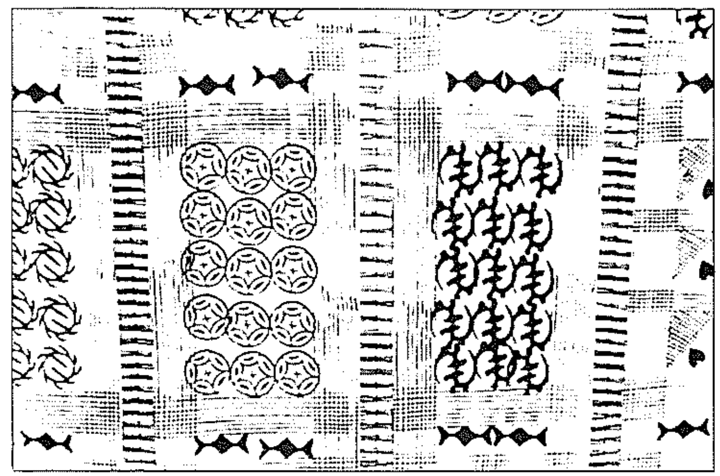
A brick wall, a checkerboard, and a tiled bathroom floor are two examples of *tessellations*, repeated patterns that completely fill a space with no overlapping. The basic design element may be a triangle, a four-sided figure, a hexagon, or a combination of several shapes. The artist Maurits Escher used irregular figures in an amazingly clever way to tessellate a surface.

Perhaps the most celebrated example of tessellations is the art associated with the Islamic religion. Originating in the Middle East early in the seventh century, Islam spread rapidly through Turkey, North



■ FIGURE 9–6 Seven border patterns, Kuba (Zaire)

■ FIGURE 9–7 Adinkra cloth, Asante (Ghana). Photograph by Sam Zaslavsky



Africa, Spain, and India, and later to other regions. Beautiful examples of Islamic art and architecture exist in many parts of the world.

Some Islamic sects forbade the depiction of human or animal forms in art. The most frequent design elements were geometric figures, floral designs, and calligraphy based on maxims from the Koran. Decorative patterns on such diverse objects as books and bowls incorporated these design elements. The inner and outer walls of mosques, palaces, and other important buildings were covered with glazed mosaic tiles in brilliant colors, arranged to form these typical designs. Patterns based entirely on construction with compass and straightedge emphasized the central role of geometry in art.

Discussion and activities

Shapes that tessellate. Pattern blocks and similar materials are an excellent medium for the exploration of tessellations. After some free play, ask students whether they can form tessellations using only triangles, only squares, only diamonds (rhombus), and so on, for each of the available shapes.

Have they found any shapes that do not tessellate? [All pattern blocks tessellate.]

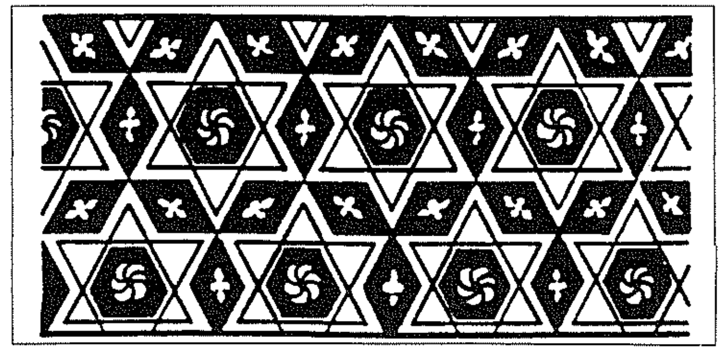
Can they form tessellations with a combination of two shapes, such as the equilateral triangle and the regular hexagon?

Students can make tracings of their tessellations with single shapes and with combinations, so that they can analyze them later. They should keep a record of their conclusions and discuss them with the class.

Angle measurement. Ask students to make a tessellation with each of the following shapes, in turn: equilateral triangles, squares, and regular hexagons. They may use pattern blocks and trace them or draw on grid paper. Use square grid paper for squares and triangular grids (isometric paper) for triangles and hexagons.

Examine one point at which all the polygons meet. What is the sum of the angles at that point? [360°.]

How many polygons meet at that point? What is the measure of each angle in the polygon? Why do these shapes tessellate? [Triangle: 6, 60°; square: 4, 90°; hexagon: 3, 120°; 60, 90, and 120 are divisors of 360.]



■ FIGURE 9–8 Islamic tessellations

Why is it not possible to tessellate with regular pentagons (five-sided polygons)? [Each angle measures 108°, which is not a divisor of 360°.]

Make a tessellation using both equilateral triangles and regular hexagons. Find the sum of the angles around any point at which several polygons meet. Is it 360°?

Create tessellations. Children can draw their original designs on identical squares or rectangles. Mount them on a large sheet of posterboard to form a tessellation (this is similar to making a patchwork quilt). On another occasion use identical parallelograms or triangles as the basic design elements.

Polygons that tessellate. Students can investigate in a systematic way the types of polygons that tessellate. For example, do all triangles tessellate, regardless of the length of their sides or the size of their angles? [Yes; the sum of their angles is 180°, and 180 is a divisor of 360.] What types of quadrilaterals tessellate? [All; the sum of their angles is 360°.]

Students should record their conclusions and be prepared to defend them by drawing illustrations or by offering a counterexample—that is, demonstrate with a shape that does not tessellate, no matter how it is rotated. This is a good project for group work. Often a student who is not considered "good in math" can offer insights based on a feeling for geometry and art.

Create tessellations. As always, students can create their own tessellations, drawing inspiration from photographs of Islamic art, from the work of Maurits Escher, or other sources. The computer can be used as a tool. See *The Mathematics of Islamic Art* (Metropolitan Museum of Art) in "Other Resources." Students should experiment with designs that can be created with only a compass and straightedge, the tools used by Islamic artists. Coloring the pattern brings out the design elements.

Chapter 10

Data Analysis & the Culture of the Community

Mathematical Topics
Computation, measurement, large numbers, fractions, percentages, averages, estimation and approximation, graphing, pattern analysis, graph theory (trees).

Cultural Connections
Students' communities.

Linked Subjects
Genealogy, environment, smoking and health, quality of life, demographics.

People are most interested in the issues that are closest to them. What better way to demonstrate the power of mathematics than to show how it helps make sense of the world and points to ways to resolve their problems?

Some issues are common to most people, while others are specific to certain communities. Often communities disagree and actually clash over the means of resolving these issues. A good example, one that involves not only school children and parents, but all branches of government—legislatures, governors, and the courts—is the question of school finances. Some schools have beautiful facilities, hundreds of computers, and well-equipped laboratories, while others are housed in decrepit buildings and have few resources. As a consequence, many low-income students are deprived of an adequate education. Equalizing the funding for all schools in this era of tight budgets generally means the Robin