The Multicultural Math Classroom

Bringing in the World

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Chapter 6

Recording & Calculating: Tallies, Knots, & Beads

Tallies on bones and wood, knots in grass and string, and beads on cords and wires have all had a place in recording and calculating with numbers. This chapter briefly reviews some aspects of the subject and suggests ways to involve students in hands-on activities.

Mathematical Topics
Counting, place value, computation, combinations, logical thinking.

Cultural Connections
Africa (ancient); Asia: China, Japan, Korea; Europe: Rome (ancient), Russia; North America: United States (contemporary); South America: Peru (Inca).

Linked Subjects
Archaeology, coding

TALLY MARKS ON THE ISHANGO BONE

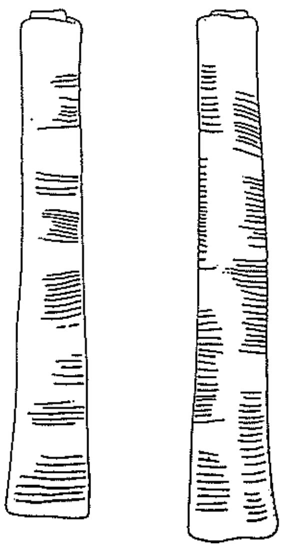
Background

The Ishango bone is one of the most fascinating and controversial of archaeological discoveries. Many millennia ago, the people of the site now called Ishango, on the shore of Lake Rutanzige in eastern Zaire, made tools for fishing and hunting. In the 1950s the Belgian archaeologist Jean de Heinzelin unearthed a number of tools that were unlike those from other African sites. Among them was a bone tool-handle marked in notches arranged in definite patterns, with a bit of quartz still fixed at its head (see Figure 6–1). De Heinzelin thought at the time that the incised bone was about 9,000 years old, a date that has since been revised, on the basis of more recent excavations, to about 20,000 B.C.E. (Before the Common Era) or even earlier.

To de Heinzelin the notches indicated a set of prime numbers and doubling of numbers. Here is the description, quoted from *Africa Counts: Number and Pattern in African Culture* (Zaslavsky 1979):

> There are three separate columns, each consisting of sets of notches arranged in distinct patterns. One column has four groups composed of eleven, thirteen, seventeen, and nineteen notches; these are the prime numbers between ten and twenty. In another column the groups consist of eleven, twenty-one, nineteen, and nine notches, in that order. The pattern here may be 10 + 1, 20 + 1, 20 − 1, and 10 − 1. The third column has the notches arranged in eight groups, in the following order: 3, 6, 4, 8, 10, 5, 5, 7. The 3 and the 6 are close together, followed by a space, then the 4 and the 8, also close together, then another space, followed by 10 and two 5's. This arrangement seems to be related to the operation of doubling. De Heinzelin concludes that the bone may have been the artifact of a people who used a number system based on ten, and who were also familiar with prime numbers and the operation of duplication. (18)

■ FIGURE 6–1 Two views of the Ishango bone



Subsequently Alexander Marshack examined the markings by microscope and came to a different conclusion. The marks were made by about thirty-nine different points, apparently at different times. He plotted the notches on the Ishango bone against a lunar model and found a close correlation. Here is possible evidence of sequential notation based on a lunar calendar for a period of almost six months. In his book *The Roots of Civilization*, Marshack (1991) discusses notational systems of similar age found in other parts of the world.

Discussion and Activities

Analysis

Students can pretend to be amateur archaeologists faced with the problem of interpreting the markings on the bone. Distribute the sketches of the Ishango bone and tell them the background, but not the interpretations by de Heinzelin or Marshack. Suggest that they count the notches in each group. (If that is too difficult, write the number next to each group before making copies of the sketches.) Can they see the relationship among the different groups of notches that de Heinzelin described? Which ancient society [Egypt] carried out multiplication by doubling?

Decision making

After they have discussed their own conclusions and compared them with those of de Heinzelin, inform them about Marshack's interpretation. Can they see any justification for it? The period of the moon's cycle is about twenty-nine and a half days. Looking at one view of the bone and adding the appropriate numbers two at a time, one obtains four different sums of thirty, the approximate length of a lunar month: 11 + 19, 21 + 9, 11 + 19 (again), and 13 + 17.

Further research

Another aspect of archaeological research is the problem of dating. Further research at the site in the 1980s led scientists to date the Ishango bone to a period thousands of years earlier than the date originally proposed. Students might investigate dating methodology.

Role play and writing

Students may want to role-play a presentation to a meeting of a scientific society or to write a newspaper article presenting their findings.

MODERN TALLIES

Background

Records in the form of tallies have been used the world over. According to British law, records of taxes were kept on tally sticks, a practice that continued from the twelfth century until 1826. As Charles Dickens tells the story, an order was finally given to burn them in a stove in the House of Lords. Unfortunately, the hot stove set fire to the paneling, which then set fire to the House of Commons. The two houses were reduced to ashes.

Tally marks are still in use in our technological culture. Look at bar codes on grocery and other items and at postal codes on the return envelopes we receive from business establishments and charitable organizations. With the postal zones encoded on the envelopes, machines can read and sort the mail, saving a great deal of time.

Each digit is represented by a combination of five long and short marks. Here is the nine-digit code on the return envelope of the Children's Defense Fund in Baltimore, Maryland 21298-9642:

Ignore the first and last bars. The tenth digit is the checking number, and can also be ignored for our purposes.

The codes for the digits zero to nine are:

0 1 2 3 4 5 6 7 8 9

Discussion and Activities

Class discussion

Ask students whether they have seen tally marks in their environment. Discuss the use of tallies in our culture and the ease with which a computer can read the codes at the checkout counter or the post office. They might bring in some examples.

Decoding postal codes

Postal codes are easier to read than bar codes. Give students some simple examples to decode, such as the following old-fashioned five-digit zip codes:

19005

10468

21297

80327

They should notice that each code has five digits and twenty-five symbols. By matching the symbols for the various digits, they should be able to construct a chart of symbols for the digits zero to nine.

Arrangements

Students should note that each symbol consists of two long and three short marks. Would some other combination of long and short tallies work as well? Three long and two short would also work. They might write out, in an orderly way, all possible arrangements of long and short marks, and count them. Combinations of one long and four short, or four long and one short, would permit only five digits. Ask students to justify their conclusions.

Analysis
Ask the students to examine their lists of symbols for 0 to 9. Are any additional combinations of two long and three short marks possible? Challenge them to prove that exactly ten arrangements, and only ten, can be formed.

Practice
Bring in coded envelopes for students to decode, or ask them to bring in such envelopes. Conceal the zip codes expressed in standard numerals, and ask students to figure them out. They can check by referring to the numerals. For ease in matching the marks to their key, they might chant in rhythm: for example, "Lo-ong, short, short, lo-ong, short."

Invention
Challenge students to invent their own codes based on tally marks. Can they improve upon the postal system?

KNOTS IN STRING: THE INCA QUIPU

Background

Every ten years the United States conducts a census of its inhabitants. From this census the Census Bureau constructs tables describing many features of life in this country—income, educational levels, number of cars and television sets, and lots more. The census figures also form the basis for the funds that the federal government allots to the cities and states for educational, health, and other programs.

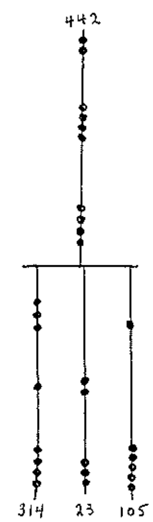
The government of the Incas (also spelled Inka) also kept elaborate records. From about the years 1400 to 1540 the Incas controlled a vast area along the west coast of South America. They imposed their institutions of government on the peoples they governed but allowed these peoples to maintain their own cultures. Cuzco, the capital, was

situated at an elevation of 11,000 feet in the land that is now Peru. The common language was Quechua, still in use today, but many different languages were spoken by the peoples of the subject lands.

The Incas maintained their control through elaborate systems of roads and information gathering. Well-trained officials in each region, as well as in the capital, kept accurate records (of things like number of people, product amounts, and tax levies) encoded in *quipus*. A quipu (KEE poo) is a collection of strings of several different colors in which the official makes knots to indicate various quantities. It has been called a "tangled mop." The knots in each string indicate a certain number in a base-ten place-value system. The strings could be detached easily for updating. Swift messengers carried them along the highways from the provinces to Cuzco and back.

Fig. 6–2 is a diagram of a simple quipu. The first string shows 314, the second shows 23, and the third shows 105. The top string shows the sum of the lower strings, 442. Note that the empty space denotes zero in our numerals. Actual quipus used a different type of knots in the units

■ **FIGURE 6–2 Quipu diagram**



place, so that there could be no confusion in reading the number. See *Code of the Quipu* (Ascher and Ascher 1981) and *Ethnomathematics* (Ascher 1991) for beautiful descriptions of quipus and quipu makers and how they functioned in Inca society.

A word of caution is in order here. Several books and articles show quipus in which each string represents a different place value, similar to an abacus. This would make the quipu very cumbersome and hard to read. In practice, each quantity was recorded on one cord. The color and placement of the cords furnished the key to the various types of data.

Many societies have used knots in fiber to keep track of time, to record quantities, and for other purposes. A young man living on the slopes of Mount Kilimanjaro in Tanzania was about to set out on a twelve-day journey. To help his wife keep track of the days, he tied twelve knots in a length of banana plant fiber. Every day she untied one knot as she awaited his homecoming. In another African culture a man would tie knots in two strings, one for himself and the other for his wife (see *Africa Counts*, Zaslavsky 1979).

Discussion and Activities

Class discussion

Ask students how they remember important events. Then discuss the types of information that are necessary to run the classroom, the school, and the home. How is this information collected? How do the various levels of government collect information, and how is this information used?

Role play

Have each group of students pretend that they are a family living in the Andes Mountains of South America. They freeze-dry potatoes to store for the future. (The potato plant originated in this region and later became a staple crop in Europe and elsewhere. At the time of the Spanish conquest, Andean farmers were producing about three thousand varieties of potato.) The local government needs to collect a record of each family's consumption of potatoes for a month. Each group should first make a table of their consumption for each week, then transfer the quantities to a diagram of a quipu.

Make a quipu

Ambitious students might take on the character of the quipu makers and devise real quipus. They might either make knots in the cords or knot small beads onto the cord. Can they carry out the addition mentally, as the quipu makers may have done?

Comparisons

Students should compare the collection and recording of data on quipus with our methods of collecting and recording data—telephone, fax, paper and pencil, computers, etc.—to make them aware of the tremendous advances in communication since the days of the Inca.

U. S. Census

Older students can learn about the census conducted every ten years. See *Counting America* (Ashabranner & Ashabranner 1989). They might discuss taking a census in the classroom, the school, or the community.

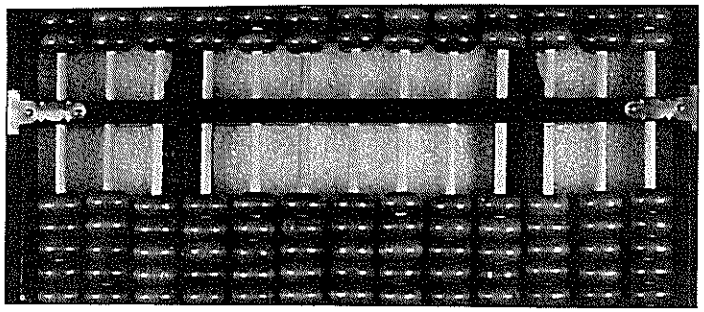
BEADS ON THE ABACUS

Background

People wonder how the Romans were able to calculate with their numerals. They didn't, not in the sense that we calculate with Indo-Arabic numerals. The Romans and other people used counting boards. This practice continued for many centuries.

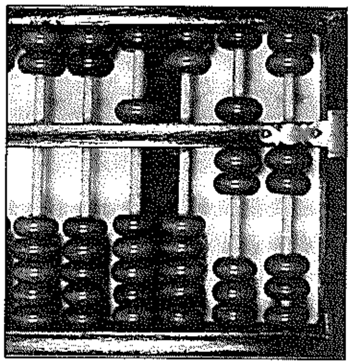
We have discussed Chinese rod numerals, originally actual sticks laid out and manipulated on a counting board. Eventually, about eight centuries ago, someone had the bright idea to string beads on cords and attach them to a frame. The result was the *suan pan,* which means "counting board" (see Fig. 6–3).

The arrangement on the *suan pan* is similar to that of Chinese rods on a counting board (see page 83). Each strand has a value that is ten times the value of the strand to its right. The *suan pan* uses a place-value system based on grouping by tens and powers of ten. As with the rod numerals, groups of five are also used. In each position the beads below the crossbar represent units from one to five. Each of the two beads above the crossbar represents a group of five.



■ FIGURE 6–3 Suan pan. Photographs by Sam Zaslavsky

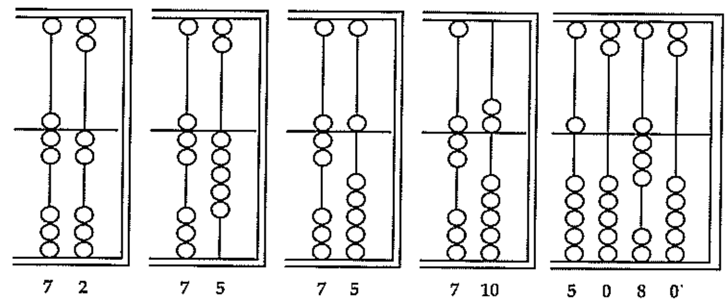
To record the number 5,072, for example:



- Start at the far right strand. Push two beads in the lower section toward the crossbar to record the 2 in the units place.
- On the second strand from the right, push two lower beads and one upper bead toward the crossbar to record 2 + 5 = 7 in the tens place.
- Leave the third strand untouched to indicate zero in the hundreds place.
- On the fourth strand, push one upper bead toward the crossbar to record 5 in the thousands place.

Now use the abacus to add 8 to 5,072:

- Think of 8 as 3 + 5. On the far right strand, push the remaining three lower beads toward the crossbar to record the addition of 3. The abacus shows 5,075.
- On the same strand, exchange the five lower beads for one upper bead. Push the lower beads down toward the frame. Push one upper bead toward the crossbar. The abacus now shows 5,075 a different way.
- On the same strand, add another five by pushing the second upper bead toward the crossbar. The abacus shows 5,070, plus 10 in the units place, for a total of 5,080.
- Exchange the two fives in the units place for one ten in the tens place. Push the two upper beads on the far right strand up to the frame. On the tens strand push another lower bead toward the crossbar. The abacus shows 5,080.



For a detailed lesson on making and using a Chinese abacus and a discussion of the mathematical aspects, see *The Language of Numbers* (Education Development Center 1994), pages 22–31.

The Japanese adopted the Chinese abacus, and, with their well-known efficiency, improved it so that fewer beads needed to be moved. In the abacus of Fig. 6–4, called the *soroban*, each strand has just four beads below the crossbar and one bead above it. The soroban in the illustration shows 5,072. Students might discuss how to add 8 to that number on the soroban.

The *suan pan* and the *soroban* are still in use and can be purchased in

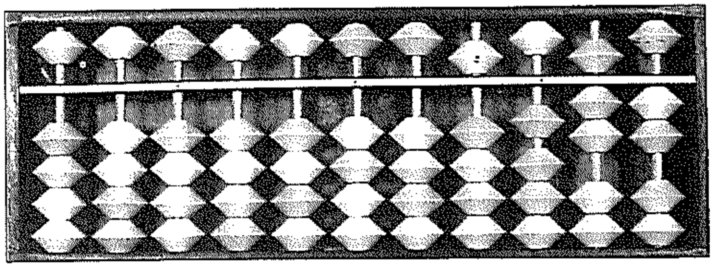


FIGURE 6–4 Soroban shows 5,072. Photograph by Sam Zaslavsky

the United States. It might seem that pushing beads on an abacus is slower than using a calculator. However, in contests between calculator operators and users of the abacus, the abacus was faster for addition and subtraction. People who use the abacus learn to do many operations mentally. In fact, Japanese children learn to use the abacus along with pencil-and-paper methods of calculation.

In the 1970s, Hang Young Pai, a Korean mathematician, introduced to the United States a method of calculating on the fingers that his father had invented in Korea. He called it *Chisanbop*; it is also known as *Fingermath*.™ The basis for the method is the Korean abacus, which is identical to the soroban. The four fingers are equivalent to the four lower beads on the soroban, and the thumb represents five. Counting and calculating start with the right hand, representing the units place, while the left hand indicates the number of tens.

The Russian abacus (Fig. 6–5), called a *scety* (s'CHAW tee), from the Russian word "to count," may be easier for American children to use. Perhaps they played with a similar device when they were small. Such devices were probably modeled on the *scety*. When the French invaded Russia in 1812, a French mathematician noticed that Russians were using this abacus. He thought it was a wonderful way for children to learn arithmetic, and brought the idea back to France. From there it spread to other European countries and to America. Even today, a *scety* often lies next to the cash register in Russian shops. The simplified *scety* illustrated here shows 5,072.03, and is handy for computing dollars and cents. Note that the two middle beads on each strand are darker, to facilitate counting the beads. The decimal point is indicated by a strand with a single bead. To record a number, slide the appropriate number of beads from right to left.

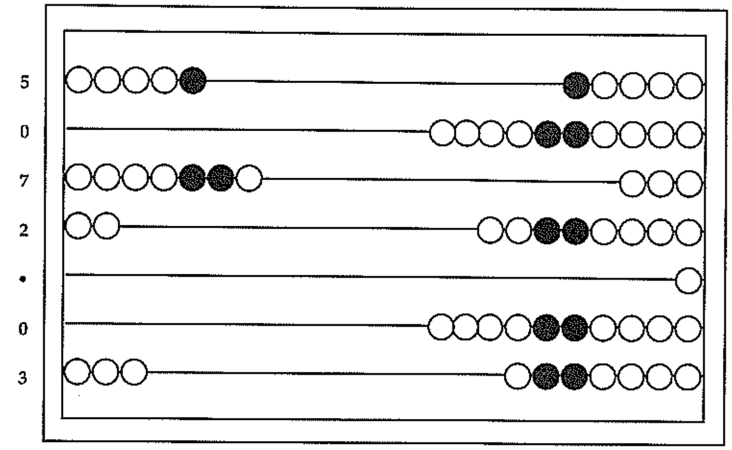


FIGURE 6–5 Scety shows 5,072.03.

Discussion and Activities

Using an abacus

Children should have access to at least one type of abacus, or they can make their own. The abacus provides an excellent way to reinforce concepts of place value and regrouping. Both the Russian and the Chinese devices permit a number to be shown in more than one way. For example, to show ten on the *scety*, either slide ten beads on the units strand to the left, or just one bead on the tens strand. Here are three ways to show ten on the *suan pan*: on the units strand move five lower beads and one upper bead; or move two upper beads; or move one lower bead on the tens strand. Why is it not possible to show ten in more than one way on the *soroban*? Ask students to carry out all the possibilities discussed here, either on an abacus or by making diagrams.

Comparison with standard numerals

Basic to any series of lessons with an abacus is to relate it to our base-ten place-value system of numerals. Discuss similarities and differences. Students should recognize that both systems use place value. For example, in the number 5,072, the digit for 5 in the thousands place means five times one thousand, or five thousand. How does the abacus show five thousand? That depends to some extent on the type of abacus. How does the abacus show zero? Can it be used to show a decimal fraction or a common fraction?

Computation

Students should try to use the abacus for addition and subtraction, starting with two one-digit numbers. Multiplication and division are more complex. Perhaps a parent or member of the community is facile with some type of abacus and can demonstrate methods of calculating. An alternative is to visit a Chinese or other East Asian restaurant that uses an abacus.

Try a different type

Once students have become accustomed to one type of abacus, they might try another type.

Teaching young children

A good exercise is for each group to make a simple abacus with two or three strands, write and illustrate a manual for its use, and teach a group of younger children the skills they have acquired.

Research and invention

Ambitious students may want to research other types of counting devices. Perhaps they will learn the *Chisanbop* method of finger calculation. Or they might invent an abacus to accommodate the Maya base-twenty system of numeration, with subgroups of fives. I actually saw a photograph of such an abacus, from the collection in a Portuguese museum, but was unable to learn more about it. It consisted of a number of horizontal strands with a divider down the middle. On each cord were four beads on one side of the divider and three beads on the other side. The cords were attached to a cloth or leather wrapping, which could then be rolled up for ease in carrying.

Chapter 7

How People Use Numbers

Theme

Use of numbers in such universal activities as trading, keeping track of time, measuring objects, and collecting data.

Mathematical Topics

Computation, mental arithmetic, equivalence, estimation, measurement systems.

Cultural Connections

Africa: Ancient Egypt, West Africa (ancient Ghana and Mali, Asante, Nigeria), Congo; Asia: Ancient Mesopotamia, India, China, Turkey (ancient Lydia), Japan, Islamic, Jewish; Europe: Spain, Norse (Viking); America: Iroquois, Aztec, Maya.

Linked Subjects

Astronomy, timekeeping.

TRADE, MONEY, AND MENTAL ARITHMETIC

Background

A long time ago the people of a community made or grew all the things they needed without having to buy or sell. Perhaps a farmer exchanged a sack of corn for an axe or a sheepskin pelt. Or the village might exchange wool from their sheep for the pots made in another village. This system of exchange is called barter. In some rural areas barter is still the main method of trading.