The Multicultural Math Classroom

Bringing in the World

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ackground

When I contracted to take an apartment on the thirteenth floor of a soon-to-be-constructed building, the agent solemnly shook my hand and congratulated me on my courage. Most tall apartment and office buildings in the United States skip thirteen when they number their floors. We even have a word, *triskaidekaphobia*, that means "fear of the number thirteen." Yet the Maya considered thirteen to be one of the most favorable numbers and devised a ritual calendar composed of thirteen months of twenty days each.

In many cultures seven is a significant number. The peoples of ancient Mesopotamia endowed the number seven with special significance that comes down to us in the form of a seven-day week. For Pythagoras and the ancient Greeks, even numbers were feminine, related to earthly things, and odd numbers were masculine, pertaining to the celestial.

iscussion and Activities

Class discussion and research
Ask students what they know about beliefs in the significance of various numbers. Have them list such beliefs and try to trace them to specific societies, if possible. They might do further research by consulting their families, neighbors, and written sources. For information, see books on the history of mathematics and the extensive treatment of significant numbers in *From One to Zero* (Ifrah 1985) and *The Mystery of Numbers* (Schimmel 1993).

Chapter 5

Numerals: Symbols for Numbers

Mathematical Topics
Systems of numerals, base, place value, zero.

Cultural Connections
Africa: Ancient Egypt, medieval North Africa; America: Maya; Asia: China, India; Europe: Greece; Middle East: Mesopotamia, Hebrew.

SYMBOLS FOR NUMBERS

Introduction

We have a wonderful method of writing numbers. In fact, this system is considered one of the most important inventions in the history of humankind. To appreciate just how wonderful it is, try to imagine doing calculations with Roman numerals. How would you represent fractions and decimals? How would you enter numbers into a calculator or computer?

Our system of numerals, written symbols for numbers, is wonderful because it has three features: place value (positional notation) for successive powers of a base number, separate symbols for the numbers from one to nine, and a symbol for zero. The system originated in India about the seventh century, although some features had existed previously. Some historians think that positional notation was borrowed from the Chinese method of placing rods in columns on a counting board. Be that as it may, merchants and scientists in the Arabic-speaking lands adopted the system. Therefore we call these symbols Indo-Arabic numerals. You may have also seen them referred to as Hindu-Arabic numerals, or just plain Arabic numerals. The word *Hindu* indicates a religion, while *Indo* refers to the country of origin. I believe Indo-Arabic is the most accurate designation.

Of course, symbols for numbers have a history going back thousands of years. By exploring some of these systems, students can compare them with one another and with the Indo-Arabic numerals they take for granted. Such comparisons lead to a better understanding of place value and computational procedures. In this chapter I discuss the following systems and how they can be incorporated into the curriculum: Egyptian hieroglyphics, the cuneiform numerals of Mesopotamia, Chinese rod numerals, and the Maya system of bars and dots in Middle America. Later in the chapter I introduce a game based on the Greek and Hebrew practice of using the letters of their alphabets as numerals.

Grace Cohen, a third-grade teacher in New York City, begins the school year with a study of different numeration systems. Although the emphasis is mainly on Roman and Egyptian numerals, children study wall charts showing other systems—Hebrew, Greek, Mayan, Chinese, and Babylonian (Mesopotamian). Here are some of the questions she poses:

- What are the possible reasons for the symbols for five and ten in the Roman system?
- Why do you think the Egyptians chose the lotus flower as the symbol for one thousand?
- How are the various systems similar and how do they differ?

Students design murals about Egyptian life and culture, integrating mathematics with art, language arts, history, and geography. They also form cuneiform symbols on clay tablets in imitation of the Babylonians. As the culmination of the unit, students often make up their own system, using some of the features they have discussed. These activities do much to remove the fear of math that some of these children have already developed.

Cohen also teaches courses for elementary teachers and discovers that few teachers are familiar with other systems of numeration. Her classes include people from China, Korea, and the Middle East, and they are always pleased to share knowledge of their number systems with the other participants. The students from these countries are quite facile in their "native" number systems, indicating that they have studied at least two systems, the Indo-Arabic and their own.

EGYPTIAN HIEROGLYPHICS

Background

"Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets."

So begins the papyrus of the scribe Ah-mose, the source of a great deal of our knowledge about ancient Egyptian mathematics. In 1858 it was purchased by Henry Rhind and later given to the British Museum. Although Ah-mose (also called Ahmes) wrote the papyrus about the year 1650 B.C.E. (Before the Common Era), the exercises he copied date to a period about two hundred years earlier. Ah-mose wrote in a script we call *hieratic*. The symbols that the Egyptians engraved on stone monuments and temple walls are named hieroglyphs.

The Ah-mose papyrus has eighty-seven problems, dealing with such subjects as calculating the amount of grain needed to make a certain quantity of bread, paying workers' wages, and calculating areas and volumes of many types of objects. Some are just "guess my number" puzzles, solvable by simple algebra.

Hieroglyphic numerals

Here are the Egyptian numerals for the powers of ten up to one million:

1 10 10^2 10^3 10^4 10^5 10^6

Main features of the system

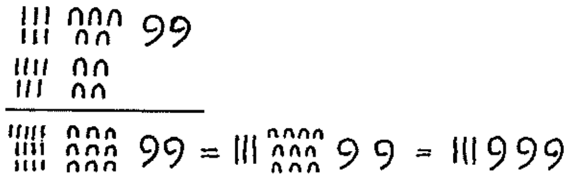
- It is based on ten and powers of ten.
- It is additive; that is, the symbol for ten is repeated three times to show thirty. The symbols might be made small to accommodate several of the same symbols in one number.
- The smallest value of the number is on the left. (Numerals were sometimes written in the opposite direction.)
- It has no symbol for zero as a placeholder. A zero symbol was unnecessary because the numbers did not have place value (positional notation). If the symbols were mixed up, we could still read the number correctly, although it might be less

convenient. However, there is evidence that the Egyptians had symbols for zero to denote the absence of a quantity or the starting point of a scale for measuring.

Addition

The need for regrouping is obvious in an addition problem like the following:

256
+ 47

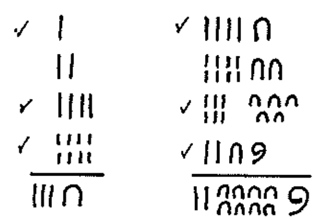


Multiplication

The ancient Egyptians multiplied two numbers by a process of doubling, a method that was still in use in Europe thousands of years later in a slightly modified form. Suppose the scribe wanted to find the product of 13 and 14:

- Set up two columns. Write the number 1 in the first column and 14 in the second column.
- Double the numbers in both columns. Continue doubling until the next number in the first column would be greater than 13, the first factor.
- Check off the numbers in the left column that add up to 13, and the corresponding numbers in the second column.
- Add the checked numbers in the right column. This sum is the product of 13 and 14.

	* 1	* 14
	2	28
	* 4	* 56
	* 8	* 112
Sums:	13	182



The doubling procedure works because any number can be expressed as the sum of powers of two:

$$13 = 1 + 4 + 8 = 2^0 + 2^2 + 2^3$$

In effect, the procedure comes down to:

$$14 \times (1 + 4 + 8) = 14 + 56 + 112.$$

Fractions

Egyptian fractions can provide a novel approach to a topic that many students find difficult. The Egyptians wrote most fractions as the sums of two or more different unit fractions, fractions having the numerator one. (The exception was the fraction two thirds.) For example, two fifths would be written as one third plus one fifteenth. The Ah-mose papyrus includes lists of such fractions.

Eye of Horus fractions

For grain measurement, however, the scribes used a different set of fractions, symbols based on the Eye of Horus:
Note that the parts represent unit fractions with denominators that are successive powers of two.

■ FIGURE 5–1 Eye of Horus fractions

Discussion and Activities

Decoding

I like to present Egyptian hieroglyphic numerals to students in the form of a puzzle for them to solve. I might give them the following information as clues:

4 37 209 3,850

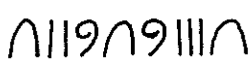
Then I ask them to decipher several hieroglyphic numerals. Of course, the examples are carefully selected to reveal the important features of the Egyptian system.

Hands-on manipulation

Students can form the symbols using pipe cleaners or similar materials and actually manipulate them. A lucky teacher in Illinois with connections to an appropriate manufacturing facility had sets of the symbols stamped out from plastic sheets. Perhaps a distributor of math materials will pick up the idea.

Place value

Challenge students to read a set of mixed-up symbols like these:



[235]

In how many ways can students write 42 in Egyptian numerals if the order of the symbols is disregarded?

By way of contrast, ask them to change the order of the symbols in the Indo-Arabic numeral 421; does the meaning remain the same, as with the Egyptian? Why is there a difference?

David Whitin presented these challenges to his third-grade class. He writes in *Read Any Good Math Lately?*: "Their comments helped to highlight a central aspect of the place value concept, the value of the place, particularly when compared with the Egyptian non-place-value system. Because the children were encouraged to discover and describe the concept in their own words rather than being given a rule, their

knowledge was grounded in their own thinking" (Whitin & Wilde 1992, 43).

Efficiency

Whitin suggests another way to contrast the two systems. In which system can the students write the number 98 more quickly? Students might be paired; one writes the Indo-Arabic numerals as many times as possible while the other writes 98 in Egyptian numerals twice. Clearly the Egyptian system is more cumbersome and less efficient except in certain instances. Can students find the exceptions (e.g., 100,000)?

Writing

Encourage groups of students to design stone monuments commemorating important events in Egyptian history, real or imaginary. They should include a specified number of Egyptian numerals, although the text may be in our Latin letters. They may want to write from right to left, Egyptian style.

Addition and subtraction

Once students have learned the system, they might add and subtract with Egyptian numerals. Include exercises that require regrouping. They should check by translating into Indo-Arabic numerals.

Multiplication

Pose some exercises in multiplication, or ask students to make up several for their classmates to work out. Can they carry out the doubling procedure using hieroglyphs? They should check by translating into Indo-Arabic numerals.

I had the good fortune to be asked to teach Egyptian math to small groups of fifth and sixth graders. After I had worked with them, they returned to their classroom and taught the others what they had learned. One girl was considered almost hopeless in math. Imagine her pride when she was able to make up several multiplication exercises, write them on a master sheet for duplicating, and teach the procedure to the class! This was a new topic for everyone, and she was not hampered by her previous failures.

Older students might be challenged to explain why the doubling procedure works. How might this procedure be adapted to perform division?

Fractions
Ask students to rewrite 2/7 and 2/9, for example, as the sums of unit fractions (2/7 = 1/4 + 1/28; 2/9 = 1/5 + 1/45 or 1/6 + 1/18).

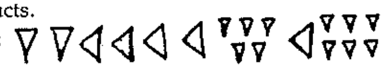
Eye of Horus fractions
How are these fractions related to the procedure for multiplication? Can you find the sum of all the Eye of Horus fractions without having to add them up? How would you show this sum on a number line?

MESOPOTAMIAN MARKS IN CLAY

Background

The fertile region known as Mesopotamia covered a territory that is roughly present-day Iraq. This region is also referred to as Babylonia, after Babylon, one of its cities. Early in its history, many thousand years ago, it had already established trade links with neighboring communities.

It is thought that written numeration began with the need to keep inventories and to record transactions. Marks made with a stylus on clay tablets, called *cuneiform* numerals, indicated quantities of various items, and were perhaps the earliest records of taxation and widescale redistribution of goods and products.

Here is the numeral for 9,916: 

Indeed, this is a puzzle!

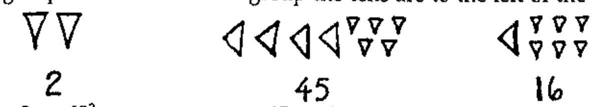
The Mesopotamians had two types of symbols:

A vertical wedge represented one: 

A horizontal wedge represented ten: 

To represent a number between one and nine, they made the corresponding number of vertical wedges on the clay tablet. For multiples of ten up to fifty, they recorded the appropriate number of horizontal wedges.

The Mesopotamians introduced place value based on grouping by sixties and powers of sixty. Below, the symbols in the Mesopotamian representation of 9,916 are grouped to show the value of each position or group. Note that in each group the tens are to the left of the ones.



2×60^2 45×60 16×1

$(2 \times 60^2) + (45 \times 60) + (16 \times 1) = (2 \times 3{,}600) + 2{,}700 + 16 = 9{,}916$

They left a space if a certain power was missing. For example:

$= (25 \times 60^2) + (0 \times 60) + 38 = 90{,}000 + 38 = 90{,}038$

Of course, this was not a very good substitute for a zero symbol. Suppose the scribe or bookkeeper was careless about the size of the space. Eventually, about 2,200 years ago, two thousand years after the invention of place-value notation, a symbol for zero—two small slanting wedges—appeared. It was used only within the numeral, not at the end. Therefore it was still possible to interpret the numeral in different ways.

can mean $(2 \times 60) + 11 = 131$ or $(2 \times 60^2) + (11 \times 60) = 7{,}860$, or sixty times 7,860, and so on. One had to judge by the context.

To summarize, the Babylonian (Mesopotamian) system of numerals involved the following features:

- Cuneiform writing—wedges made by a stylus in clay.
- Two types of grouping, by sixties and by tens.
- Two types of symbols: a vertical wedge and a horizontal wedge.
- Place value based on sixty and powers of sixty.
- A symbol for zero used only within a numeral.

Students might wonder why sixty was selected as the unit for grouping. One reason is that certain measures consisted of sixty smaller measures. Note that sixty is divisible by many numbers: two, three, four, five, six, ten, twelve, fifteen, twenty, and thirty, making it easy to divide a large measure into many different fractional parts.

How did the Babylonians carry out calculations with so complex a system of numerals? Some clay tablets consist of tables for multiplication and division, tables of squares, and tables of cubes, indicating that they needed to calculate with such quantities.

Discussion and Activities

Class discussion
Ask students to name groupings in our culture that relate to sixty. We have sixty minutes in an hour and sixty seconds in a minute, as well as multiples and fractions of sixty, such as 360 degrees in the circle, twelve units in a dozen, twelve inches in a foot, and many more.

Translate and calculate
Encourage students to translate numbers into cuneiform symbols and cuneiform symbols into Indo-Arabic numerals. They might try to add and subtract with these numerals.

Role play
Students might pretend to be merchants and buyers and write the numbers involved in their transactions in cuneiform on clay tablets. Each group might write a scenario for such a role play and perform it for the class.

CHINESE ROD NUMERALS

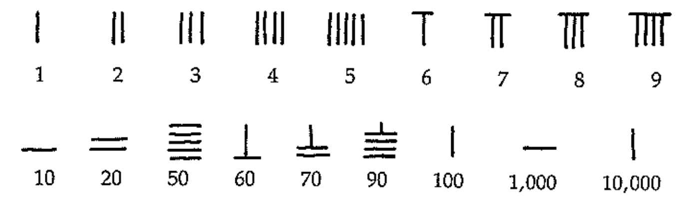
Background

After the complexities of the Mesopotamian (Babylonian) system of numeration based on grouping by sixties and tens, Chinese rod numerals, also called stick numerals, will seem easy. They had their origin in the Chinese counting board, a table on which bamboo sticks were arranged in columns based on grouping by powers of ten.

The Chinese actually had four different sets of numerals: basic, official, commercial, and rods or sticks. Long before Indo-Arabic numerals became common, the Chinese devised a set of distinct symbols

for the numbers from one to nine, which they then combined with symbols for ten, hundred, thousand, and so forth, to write any numerals they pleased. We shall concentrate on the rod numerals, which date to about 400 B.C.E. They are the easiest to work with and are ideal for younger children.

Here are some of the numerals. Since they follow a pattern, it should not be difficult to fill in the missing numbers.

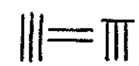


The numerals above represent bamboo sticks laid out on a table in columns with headings one, ten, hundred, thousand, and so on. An empty column stood for zero. Usually the headings were not explicit; the alternating directions of the rods indicated the values of the columns. The rods were manipulated to carry out computations. Often two sets of rods of different colors, usually red and black, represented positive and negative numbers.

About the thirteenth century the Chinese borrowed the zero symbol from India for the written form of the rod numerals. Here is the written symbol for 3,702: 

Discussion and Activities

Decoding
I like to present a numeral and ask students to guess the value:

 [328]

I offer some hints: it has three digits, place value (like ours), and each digit is related to the number of fingers on one or two hands. An alternative strategy is to identify the number above, and then ask students to figure out the value of other Chinese stick numerals.

Hands-on manipulation
Students might make a counting board having columns headed "one," "ten," and so on.

10,000	1,000	100	10	1

After they have learned to form a variety of numerals with toothpicks on the counting board, they can carry out transactions involving addition and subtraction. To add or subtract, they move and regroup the toothpicks. They should write out the scenarios for this role play.

Translation
Have students copy their stick numerals in the previous activity on diagrams of the counting board and translate them into Indo-Arabic numerals. These diagrams might form the basis for attractive posters.

Research
Investigate how the Chinese used positive and negative rod numerals.

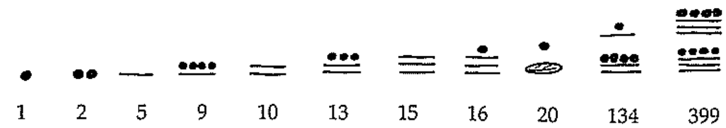
BARS AND DOTS OF THE MAYA

Background

The Maya and their predecessors have lived, and still live, in southern Mexico and northern Central America, a region often called Mesoamerica (Middle America). At least two thousand years ago they were engraving their numerals in stone monuments that told the history of their rulers and other important events. Like their spoken number words, the system is based on grouping by fives and twenties. Numerals are written in vertical columns. The system has place value, with the smallest value at the bottom of the column. With two symbols, a bar and a dot, and a symbol for zero that looked like a shell, the Maya were able to denote the largest numbers. In the sixteenth century the Spanish conquerors told of Maya books containing intricate astronomical

calculations, but few survived destruction either by the conquerors or by missionaries, who considered them works of the devil. Only in the last few years have these documents and monuments been translated.

Below are several numerals and their translation. The system is completely predictable; once you know the procedure, you can write any numeral.



We know that for everyday calculations the Maya laid out sticks and pebbles on the ground or on a table. Perhaps that's how the written symbols originated. Unfortunately we don't know what procedures they used to arrive at the answers to astronomical calculations. In one instance a date one-and-a-quarter million years in the past was calculated.

Discussion and Activities

Hands-on manipulation
Students can work with toothpicks and beans to lay out the numerals and perform calculations, and then transfer their work to paper and pencil. Can they translate from Indo-Arabic numerals to Maya, or the reverse, using only mental arithmetic?

Grouping
Students should write Maya numbers in our numerals to show the grouping:

$$= (6 \times 1) + (14 \times 20) + (5 \times 20^2) =$$
$$(6 \times 1) + (14 \times 20) + (5 \times 400) =$$
$$6 + 280 + 2{,}000 = 2{,}286$$

Research
Students may want to research the various interrelated calendars that the Maya used, and learn how they adapted the system of numerals to calculate the number of 360-day "years" between events, as well as the amount of time that had elapsed from the beginning of their reckoning in the year we call 3114 B.C.E.