

## Three-Part Invention

*Achilles (a Greek warrior, the fleetest of foot of all mortals) and a Tortoise are standing together on a dusty runway in the hot sun. Far down the runway, on a tall flagpole, there hangs a large rectangular flag. The flag is solid red, except where a thin ring-shaped hole has been cut out of it, through which one can see the sky.*

*Achilles:* What is that strange flag down at the other end of the track? It reminds me somehow of a print by my favorite artist, M. C. Escher.

*Tortoise:* That is Zeno's flag.

*Achilles:* Could it be that the hole in it resembles the holes in a Möbius strip Escher once drew? Something is wrong about that flag, I can tell.

*Tortoise:* The ring which has been cut from it has the shape of the numeral for zero, which is Zeno's favorite number.

*Achilles:* But zero hasn't been invented yet! It will only be invented by a Hindu mathematician some millennia hence. And thus, Mr. T, my argument proves that such a flag is impossible.

*Tortoise:* Your argument is persuasive, Achilles, and I must agree that such a flag is indeed impossible. But it is beautiful anyway, is it not?

*Achilles:* Oh, yes, there is no doubt of its beauty.

*Tortoise:* I wonder if its beauty is related to its impossibility. I don't know; I've never had the time to analyze Beauty. It's a Capitalized Essence; and I never seem to have the time for Capitalized Essences.

*Achilles:* Speaking of Capitalized Essences, Mr. T, have you ever wondered about the Purpose of Life?

*Tortoise:* Oh, heavens, no.

*Achilles:* Haven't you ever wondered why we are here, or who invented us?

*Tortoise:* Oh, that is quite another matter. We are inventions of Zeno (as you will shortly see); and the reason we are here is to have a footrace.

*Achilles:* A footrace? How outrageous! Me, the fleetest of foot of all mortals, versus you, the ploddingest of all plodders! There can be no point to such a race.

*Tortoise:* You might give me a head start.

*Achilles:* It would have to be a huge one.

*Tortoise:* I don't object.

*Achilles:* But I will catch you, sooner or later—most likely sooner.

*Tortoise:* Not if things go according to Zeno's paradox, you won't. Zeno is hoping to use our footrace to show that motion is impossible, you see. It is only in the mind that motion seems possible, according to Zeno. In truth, Motion Is Inherently Impossible. He proves it quite elegantly.

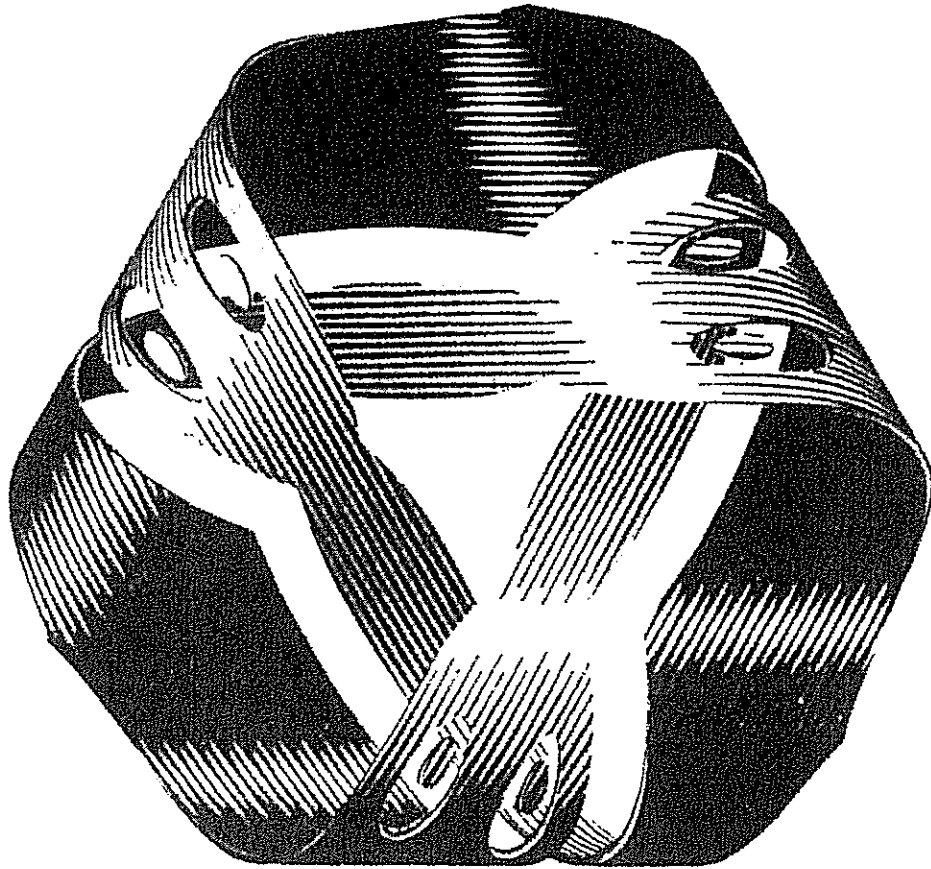


FIGURE 10. Möbius Strip I, by M. C. Escher (wood-engraving printed from four blocks, 1961).

*Achilles:* Oh, yes, it comes back to me now: the famous Zen kōan about Zen Master Zeno. As you say, it is very simple indeed.

*Tortoise:* Zen kōan? Zen Master? What do you mean?

*Achilles:* It goes like this: Two monks were arguing about a flag. One said, "The flag is moving." The other said, "The wind is moving." The sixth patriarch, Zeno, happened to be passing by. He told them, "Not the wind, not the flag; mind is moving."

*Tortoise:* I am afraid you are a little befuddled, Achilles. Zeno is no Zen master; far from it. He is, in fact, a Greek philosopher from the town of Elea (which lies halfway between points A and B). Centuries hence, he will be celebrated for his paradoxes of motion. In one of those paradoxes, this very footrace between you and me will play a central role.

*Achilles:* I'm all confused. I remember vividly how I used to repeat over and over the names of the six patriarchs of Zen, and I always said, "The sixth patriarch is Zeno, the sixth patriarch is Zeno . . ." (*Suddenly a soft warm breeze picks up.*) Oh, look, Mr. Tortoise—look at the flag waving! How I love to watch the ripples shimmer through its soft fabric. And the ring cut out of it is waving, too!

*Tortoise:* Don't be silly. The flag is impossible, hence it can't be waving.  
The wind is waving.

*(At this moment, Zeno happens by.)*

*Zeno:* Hallo! Hulloo! What's up? What's new?

*Achilles:* The flag is moving.

*Tortoise:* The wind is moving.

*Zeno:* O Friends, Friends! Cease your argumentation! Arrest your vitriolics! Abandon your discord! For I shall resolve the issue for you forthwith. Ho! And on such a fine day!

*Achilles:* This fellow must be playing the fool.

*Tortoise:* No, wait, Achilles. Let us hear what he has to say. Oh, Unknown Sir, do impart to us your thoughts on this matter.

*Zeno:* Most willingly. Not the wind, not the flag—neither one is moving, nor is anything moving at all. For I have discovered a great Theorem, which states: "Motion Is Inherently Impossible." And from this Theorem follows an even greater Theorem—Zeno's Theorem: "Motion Unexists."

*Achilles:* "Zeno's Theorem"? Are you, sir, by any chance, the philosopher Zeno of Elea?

*Zeno:* I am indeed, Achilles.

*Achilles (scratching his head in puzzlement):* Now how did he know my name?

*Zeno:* Could I possibly persuade you two to hear me out as to why this is the case? I've come all the way to Elea from point A this afternoon, just trying to find someone who'll pay some attention to my closely honed argument. But they're all hurrying hither and thither, and they don't have time. You've no idea how disheartening it is to meet with refusal after refusal. Oh, but I'm sorry to burden you with my troubles. I'd just like to ask one thing: Would the two of you humor a silly old philosopher for a few moments—only a few, I promise you—in his eccentric theories?

*Achilles:* Oh, by all means! Please do illuminate us! I know I speak for both of us, since my companion, Mr. Tortoise, was only moments ago speaking of you with great veneration—and he mentioned especially your paradoxes.

*Zeno:* Thank you. You see, my Master, the fifth patriarch, taught me that reality is one, immutable, and unchanging; all plurality, change, and motion are mere illusions of the senses. Some have mocked his views; but I will show the absurdity of their mockery. My argument is quite simple. I will illustrate it with two characters of my own Invention: Achilles (a Greek warrior, the fleetest of foot of all mortals), and a Tortoise. In my tale, they are persuaded by a passerby to run a footrace down a runway towards a distant flag waving in the breeze. Let us assume that, since the Tortoise is a much slower runner, he gets a head start of, say, ten rods. Now the race begins. In a few bounds, Achilles has reached the spot where the Tortoise started.

*Achilles:* Hah!

*Zeno:* And now the Tortoise is but a single rod ahead of Achilles. Within only a moment, Achilles has attained that spot.

*Achilles:* Ho ho!

*Zeno:* Yet, in that short moment, the Tortoise has managed to advance a slight amount. In a flash, Achilles covers that distance, too.

*Achilles:* Hee hee hee!

*Zeno:* But in that very short flash, the Tortoise has managed to inch ahead by ever so little, and so Achilles is still behind. Now you see that in order for Achilles to catch the Tortoise, this game of "try-to-catch-me" will have to be played an INFINITE number of times—and therefore Achilles can NEVER catch up with the Tortoise!

*Tortoise:* Heh heh heh heh!

*Achilles:* Hmm . . . hmm . . . hmm . . . hmm . . . Hmm . . . That argument sounds wrong to me. And yet, I can't quite make out what's wrong with it.

*Zeno:* Isn't it a teaser? It's my favorite paradox.

*Tortoise:* Excuse me, Zeno, but I believe your tale illustrates the wrong principle, does it not? You have just told us what will come to be known, centuries hence, as Zeno's "Achilles paradox", which shows (ahem!) that Achilles will never catch the Tortoise; but the proof that Motion Is Inherently Impossible (and thence that Motion Unexists) is your "dichotomy paradox", isn't that so?

*Zeno:* Oh, shame on me. Of course, you're right. That's the one about how, in getting from A to B, one has to go halfway first—and of that stretch one also has to go halfway, and so on and so forth. But you see, both those paradoxes really have the same flavor. Frankly, I've only had one Great Idea—I just exploit it in different ways.

*Achilles:* I swear, these arguments contain a flaw. I don't quite see where, but they cannot be correct.

*Zeno:* You doubt the validity of my paradox? Why not just try it out? You see that red flag down there, at the far end of the runway?

*Achilles:* The impossible one, based on an Escher print?

*Zeno:* Exactly. What do you say to you and Mr. Tortoise racing for it, allowing Mr. T a fair head start of, well, I don't know—

*Tortoise:* How about ten rods?

*Zeno:* Very good—ten rods.

*Achilles:* Any time.

*Zeno:* Excellent! How exciting! An empirical test of my rigorously proven Theorem! Mr. Tortoise, will you position yourself ten rods upwind?

*(The Tortoise moves ten rods closer to the flag.)*

Are you both ready?

*Tortoise and Achilles:* Ready!

*Zeno:* On your mark! Get set! Go!

# Sonata for Unaccompanied Achilles

*The telephone rings; Achilles picks it up.*

*Achilles:* Hello, this is Achilles.

*Achilles:* Oh, hello, Mr. T. How are you?

*Achilles:* A torticollis? Oh, I'm sorry to hear it. Do you have any idea what caused it?

*Achilles:* How long did you hold it in that position?

*Achilles:* Well, no wonder it's stiff, then. What on earth induced you to keep your neck twisted that way for so long?

*Achilles:* Wondrous many of them, eh? What kinds, for example?

*Achilles:* What do you mean, "phantasmagorical beasts"?

FIGURE 14. Mosaic II, by M. C. Escher (lithograph, 1957).



*Achilles:* Wasn't it terrifying to see so many of them at the same time?

*Achilles:* A guitar!? Of all things to be in the midst of all those weird creatures. Say, don't you play the guitar?

*Achilles:* Oh, well, it's all the same to me.

*Achilles:* You're right; I wonder why I never noticed that difference between fiddles and guitars before. Speaking of fiddling, how would you like to come over and listen to one of the sonatas for unaccompanied violin by your favorite composer, J. S. Bach? I just bought a marvelous recording of them. I still can't get over the way Bach uses a single violin to create a piece with such interest.

*Achilles:* A headache too? That's a shame. Perhaps you should just go to bed.

*Achilles:* I see. Have you tried counting sheep?

*Achilles:* Oh, oh, I see. Yes, I fully know what you mean. Well, if it's THAT distracting, perhaps you'd better tell it to me, and let me try to work on it, too.

*Achilles:* A word with the letters 'A', 'D', 'A', 'C' consecutively inside it . . . Hmm . . . What about "abracadabra"?

*Achilles:* True, "ADAC" occurs backwards, not forwards, in that word.

*Achilles:* Hours and hours? It sounds like I'm in for a long puzzle, then. Where did you hear this infernal riddle?

*Achilles:* You mean he looked like he was meditating on esoteric Buddhist matters, but in reality he was just trying to think up complex word puzzles?

*Achilles:* Aha!—the snail knew what this fellow was up to. But how did you come to talk to the snail?

*Achilles:* Say, I once heard a word puzzle a little bit like this one. Do you want to hear it? Or would it just drive you further into distraction?

*Achilles:* I agree—can't do any harm. Here it is: What's a word that begins with the letters "HE" and also ends with "HE"?

*Achilles:* Very ingenious—but that's almost cheating. It's certainly not what I meant!

*Achilles:* Of course you're right—it fulfills the conditions, but it's a sort of "degenerate" solution. There's another solution which I had in mind.

*Achilles:* That's exactly it! How did you come up with it so fast?

*Achilles:* So here's a case where having a headache actually might have helped you, rather than hindering you. Excellent! But I'm still in the dark on your "ADAC" puzzle.

*Achilles:* Congratulations! Now maybe you'll be able to get to sleep! So tell me, what IS the solution?

*Achilles:* Well, normally I don't like hints, but all right. What's your hint?

*Achilles:* I don't know what you mean by "figure" and "ground" in this case.

*Achilles:* Certainly I know *Mosaic II*! I know ALL of Escher's works. After all, he's my favorite artist. In any case, I've got a print of *Mosaic II* hanging on my wall, in plain view from here.

*Achilles:* Yes, I see all the black animals.

*Achilles:* Yes, I also see how their “negative space”—what’s left out—defines the white animals.

*Achilles:* So THAT’S what you mean by “figure” and “ground”. But what does that have to do with the “ADAC” puzzle?

*Achilles:* Oh, this is too tricky for me. I think I’m starting to get a headache.

*Achilles:* You want to come over now? But I thought—

*Achilles:* Very well. Perhaps by then I’ll have thought of the right answer to YOUR puzzle, using your figure-ground hint, relating it to MY puzzle.

*Achilles:* I’d love to play them for you.

*Achilles:* You’ve invented a theory about them?

*Achilles:* Accompanied by what instrument?

*Achilles:* Well, if that’s the case, it seems a little strange that he wouldn’t have written out the harpsichord part, then, and had it published as well.

*Achilles:* I see—sort of an optional feature. One could listen to them either way—with or without accompaniment. But how would one know what the accompaniment is supposed to sound like?

*Achilles:* Ah, yes, I guess that it is best, after all, to leave it to the listener’s imagination. And perhaps, as you said, Bach never even had any accompaniment in mind at all. Those sonatas seem to work very well indeed as they are.

*Achilles:* Right. Well, I’ll see you shortly.

*Achilles:* Good-bye, Mr. T.